Math 1496 - Calc I

Precalculus

If we consider 2 sets X & Y and elements $x \in X$ & $y \in Y$ then we consider a relationship between x & y & would be given by the order pair (x, y).

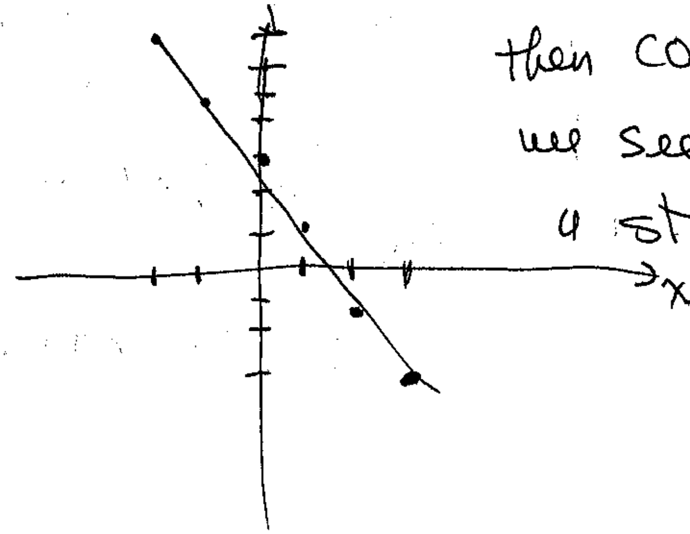
For example suppose we knew that

$$2x + y = 3 \qquad (1)$$

One way to visualize this relationship is a graph of the order pairs that satisfy (1)

If we create a table of values & plot pts

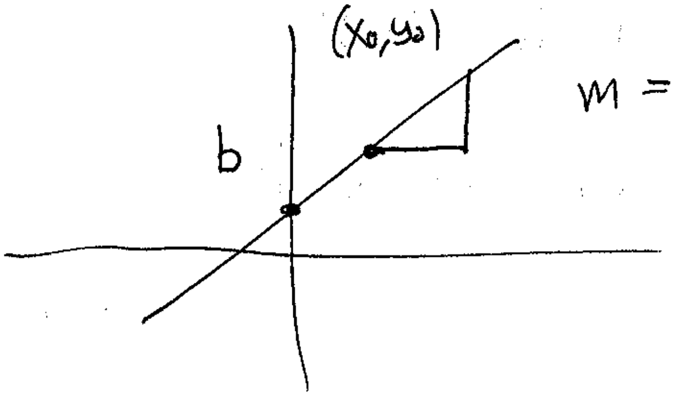
x	y
-2	7
-1	5
0	3
1	1
2	-1
3	-3



then connect the pt we see we have a straight line

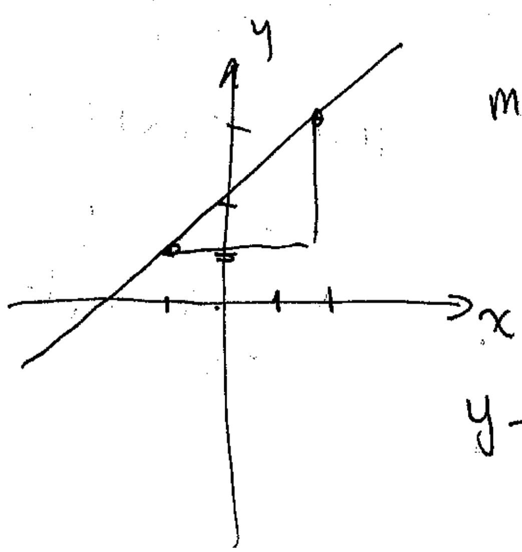
Defn general linear (straight line) eqn have eqn of the form

	form
$y - y_0 = m(x - x_0)$	pt-slope
$y = mx + b$	slope-intercept
$ax + by + c = 0$	general



$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

Ex Find the eqn of the line through $P(-1,1)$ & $Q(2,3)$



$$m = \frac{\Delta y}{\Delta x} = \frac{3-1}{2-(-1)} = \frac{2}{3}$$

$$(x_0, y_0) = (-1, 1)$$

$$y - 1 = \frac{2}{3}(x+1)$$ pt slope form

a $y - 1 = \frac{2}{3}x + \frac{2}{3}$

$y = \frac{2x}{3} + 5/3$ slope intercept (2)

$2x - 3y + 5 = 0$ general form (3)

It's a easy matter to check the Q(2,3) also satisfies our eqn

For example sub Q in (2) gives

$$y = \frac{2}{3}(2) + \frac{5}{3}$$
$$= \frac{4}{3} + \frac{5}{3} = 9/3 = 3 \checkmark$$

Given 2 lines

$$y = m_1 x + b_1, \quad y = m_2 x + b_2$$

if $m_1 = m_2$ the lines are parallel

if $m_1 \cdot m_2 = -1$ the lines are perpendicular

Quadratic Eq^n's

These are of the form

$y = ax^2 + bx + c$ a, b, c #'s

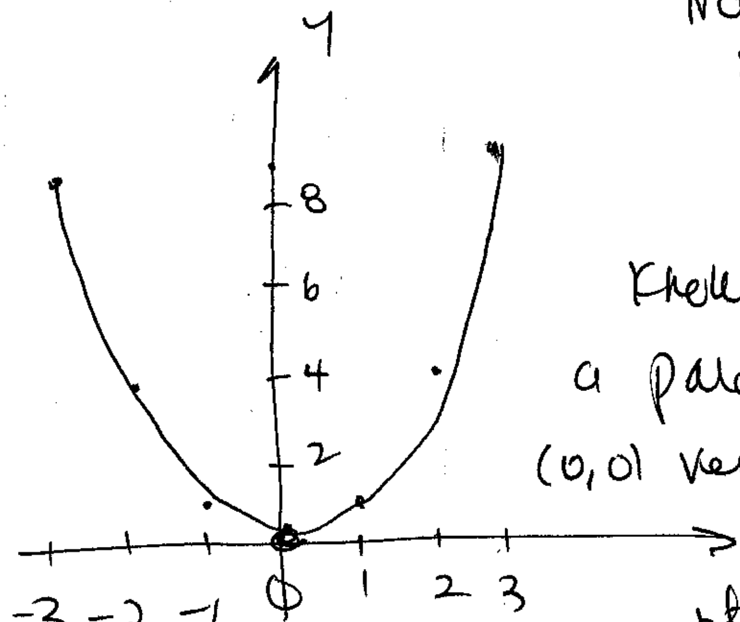
Probably the easiest is

$y = x^2$

"not to scale"

T of V

x	y
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



Known as a parabola

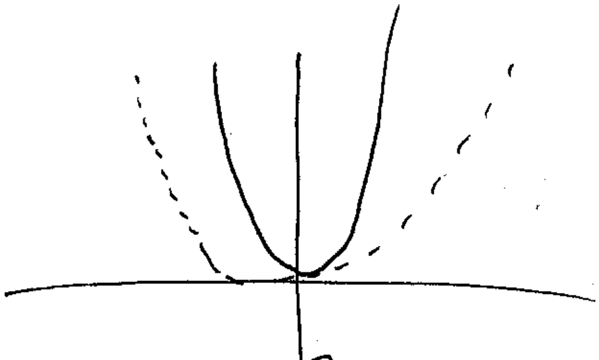
(0,0) vertex

lowest of the curve

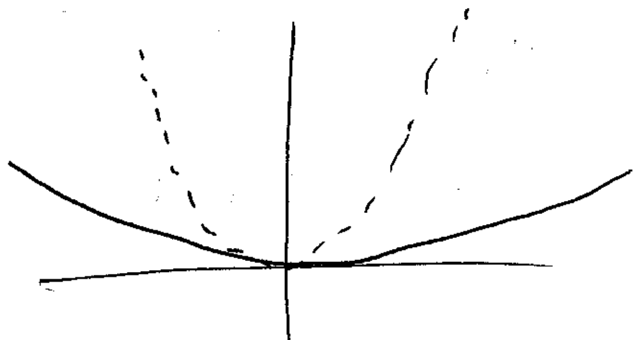
so what would the sketch (graph) of

$y = 2x^2$ or $y = \frac{1}{3}x^2$

look like?



$y = 2x^2$



$y = \frac{1}{3}x^2$

so what about

$$y = 2x^2 - 4x + 3$$

↑ ↑ now we have extra terms

<u>completing the square</u>

we know that

$$(x+k)^2 = (x+k)(x+k) = x^2 + 2kx + k^2$$

so we use this to rewrite our q-eqn slightly different

$$\begin{aligned} 2(x^2) - 4x + 3 &= 2(x^2 - 2x) + 3 \\ &= 2(x^2 - 2x + 1 - 1) + 3 \\ &= 2(x^2 - 2x + 1) - 2 + 3 \\ &= 2(x-1)^2 + 1 \end{aligned}$$

so why is this form

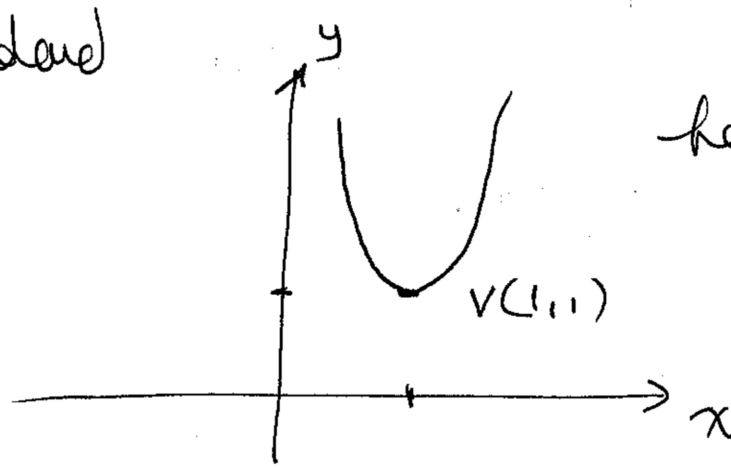
$y = 2(x-1)^2 + 1$ better

Well 1st the smallest y can be is 1

$\because (x-1)^2 \geq 0$ & $= 0$ when $x = 1$

so the vertex is located at (1,1)

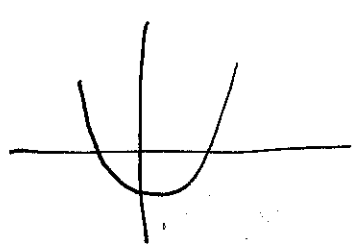
and the standard graph just move → ↑



here's the graph

<u>A Few Things about graphs</u>

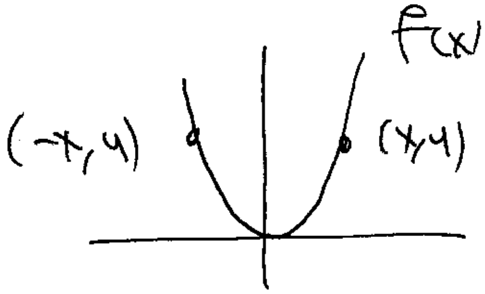
x intercepts



where f crosses x axis

y intercept - when f crosses y axis

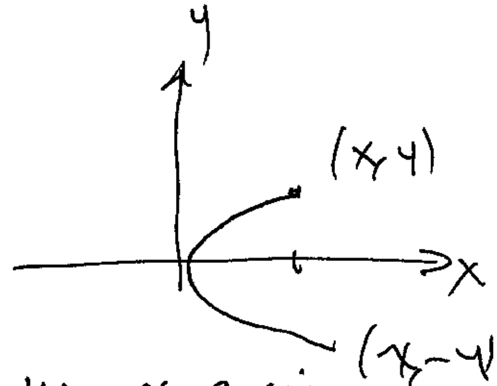
Symmetry



$y = x^2$

in y axis

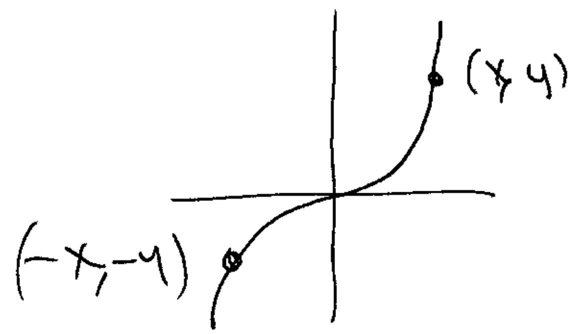
$f(-x) = f(x)$



$x = y^2$

in x axis

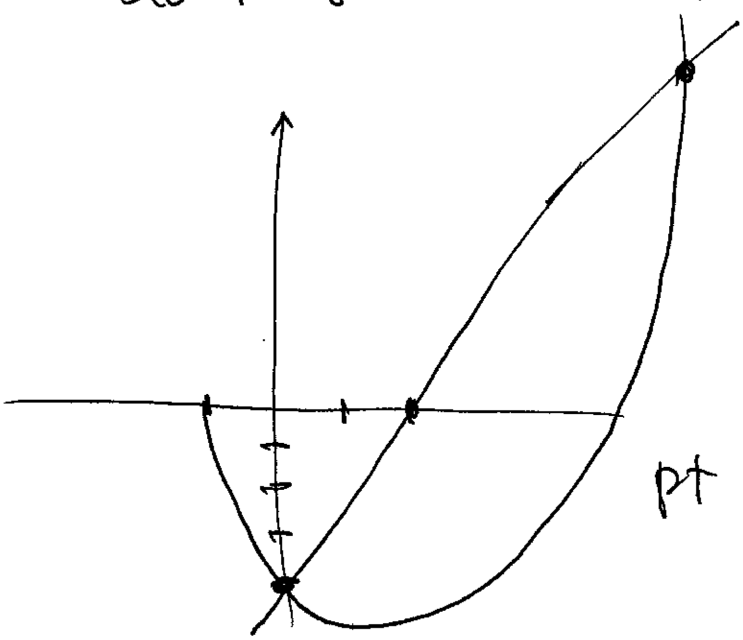
in origin



Ex $y = x^3$

pts of intersection

consider $y = x^2 - 3x - 4$, $y = 2x - 4$ ← x intercept 2, y inter -4



set eq'n =

$x^2 - 3x - 4 = 2x - 4$

$x^2 - 5x = 0$

$x(x-5) = 0$ $x = 0, 5$

pt $(0,-4)$ $(5,6)$