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Arithmetic Average Return versus Geometric Average Return: Which One is Better?

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Abstract

This case deals with the calculation of investment returns. It teaches students about arithmetic and geometric average returns. Students will learn how to compute these measures and they will also learn when to use these measures. First, they will solve two puzzling examples. After that, they will evaluate an investor's investment return. This is a hands-on practice for students who want to learn about different measures of investment returns.

Keywords: return, investment, arithmetic average, geometric average
JEL classifications: G11, G17

Introduction

Kevin is an undergraduate student at a public university. He is majoring in finance. His midterm exam is coming, and in the exam, there will be questions from investment returns. One of the topics is different return measures like arithmetic average return and geometric average return.

He was thinking that these were easy topics so he has not studied for these topics yet. Now, the exam is just two days away and he is finally looking at the chapter. In the evening, he meets with one of his classmates to study together at the university library. He was feeling very relaxed until his classmate showed him a puzzling example. The example illustrates the differences between airthmetic average return and geometric average return.

"Hey, Mike. I was feeling comfortable with these topics until you showed me this example. I don't understand this. The formulas are clear for both measures, so I enter the numbers into these two formulas, but somehow I find different answers. They are so different, it makes me crazy." Kevin exclaimed.

Mike responds "Yea, I know. Kind of strange, don't you think? I am kind of scared right now because in the exam, if I see such a question, I will just freeze, I think."

"I completely agree. We need to learn these topics. Plus I need to learn how to compute returns for my dad. Last week, he asked me how his portfolio performed over the last five years and I could not answer" Kevin adds.

"Do you think we need to look it up on the internet? Maybe some websites explain these concepts better. What do you think?" Mike asks.

"Sure, we can do that. I think we need to hurry up. Let's search the web to see if they have some explanations and examples" Kevin suggests.

Arithmetic Average and Geometric Average Returns

The two friends have found some explanations and examples on the web. The formulas for arithmetic average and geometric average returns are:

Arithmetic average return = $(R_1+R_2+\ldots+R_N)/N$

Geometric average return = $[(1+R_1)(1+R_2)\ldots(1+R_N)]^{1/N} - 1$

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where
R_1 is the first period's return,
R_2 is the second period's return,
R_N is the n^{th} period's return, and
N is the number of periods.

If we know the percentage return in each period, we can use the above formulas to compute the average and the geometric average annual returns. If we do not know the percentage returns for each period, but instead know the prices, we need to first compute the percentage returns for each period using the prices. Then, we can use those computed percentage returns to calculate the arithmetic and the geometric average annual returns using the above formulas.

For example, if a stock's price is given at Year 0, Year 1, and Year 2 as $10, $11, and $11.55 respectively, we can compute the percentage return as follows:

R_1 = % return in the first year = $(P_1-P_0)/P_0 = (11-10)/10 = 0.10 = 10\%$

For the second year, we just modify this formula like this:
R_2 = % return in the second year = $(P_2-P_1)/P_1 = (11.55-11)/11 = 0.05 = 5\%$

After we find the percentage returns in each of the two years (i.e. R_1 and R_2), we can then compute the arithmetic and the geometric average returns using the above formulas:

Arithmetic average return = $(R_1+R_2+\ldots+R_N)/N = (10+5)/2 = 7.5\%$/year
Geometric average return = $[(1+R_1)(1+R_2)\ldots(1+R_N)]^{1/N} - 1$

$= [(1+0.10)(1+0.05)]^{1/2} - 1$

$= [(1.10)(1.05)]^{0.5} - 1$

$= (1.155)^{0.5} - 1$

$= 1.0747 - 1$

$= 0.0747$

$= 7.47\%$/year

Investopedia.com explains arithmetic average and geometric average returns as follows:
"An arithmetic average is the sum of a series of numbers divided by the count of that series of numbers.

If you were asked to find the class (arithmetic) average of test scores, you would simply add up all the test scores of the students, and then divide that sum by the number of students. For example, if five students took an exam and their scores were 60%, 70%, 80%, 90% and 100%, the arithmetic class average would be 80%.

This would be calculated as: (0.6 + 0.7 + 0.8 + 0.9 + 1.0) / 5 = 0.8.
The reason you use an arithmetic average for test scores is that each test score is an independent event. If one student happens to perform poorly on the exam, the next student's

chances of doing poor (or well) on the exam isn't affected. In other words, each student's score is independent of the all other students' scores. However, there are some instances, particularly in the world of finance, where an arithmetic mean is not an appropriate method for calculating an average.

Consider your investment returns, for example. Suppose you have invested your savings in the stock market for five years. If your returns each year were 90%, 10%, 20%, 30% and -90%, what would your average return be during this period? Well, taking the simple arithmetic average, you would get an answer of 12%. Not too shabby, you might think.

However, when it comes to annual investment returns, the numbers are not independent of each other. If you lose a ton of money one year, you have that much less capital to generate returns during the following years, and vice versa. Because of this reality, we need to calculate the geometric average of your investment returns in order to get an accurate measurement of what your actual average annual return over the five-year period is.

To do this, we simply add one to each number (to avoid any problems with negative percentages). Then, multiply all the numbers together, and raise their product to the power of one divided by the count of the numbers in the series. And you're finished - just don't forget to subtract one from the result!

That's quite a mouthful, but on paper it's actually not that complex. Returning to our example, let's calculate the geometric average: Our returns were 90%, 10%, 20%, 30% and -90%, so we plug them into the formula as $[(1.9 \times 1.1 \times 1.2 \times 1.3 \times 0.1)^{1/5}] - 1$. This equals a geometric average annual return of -20.08%. That's a heck of a lot worse than the 12% arithmetic average we calculated earlier, and unfortunately it's also the number that represents reality in this case.

It may seem confusing as to why geometric average returns are more accurate than arithmetic average returns, but look at it this way: if you lose 100% of your capital in one year, you don't have any hope of making a return on it during the next year. In other words, investment returns are not independent of each other, so they require a geometric average to represent their mean."

Another website (educ.jmu.edu) explains the geometric average returns as follows:

"A geometric mean return is an average return that considers compounding and is the standard metric for conveying return performance for investments. When investment professionals refer to the average annual return, they are referring to the geometric average annual return".

The Decision

1. Kevin finally understands the concepts of arithmetic average return and geometric average return. But, one of his classmates showed him an example on this topic and he has since been puzzled.

   In the example that was given by his classmate, an investor buys a particular stock for $100. Unfortunately, the first year he owns it, the price falls to $50. The second year he owns it, the price rises back to $100, leaving him where he started

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(no dividends were paid). In the example, the question was "Without making any calculations, what was the investor's average return on this investment?"

2. The example continues with additional questions. "What is the arithmetic average return. Is it a meaningful number?"

3. Then, the next question is "What is the geometric average return? Is it a meaningful number?"

4. Kevin is really puzzled by these questions. He finds that the arithmetic average return and the geometric average return are very different. He is wondering why they are so different. Why do you think the two measures of return are so different?

5. After working on these questions, Kevin looks at his textbook for more examples. He finds an interesting example. This one he thinks is also puzzling. The example says "If we invested $10,000 five years ago and now this investment is worth $20,000, what was our arithmetic average annual returns? Can we find it?"

6. Kevin continues with the same example. It says "What was our geometric average return? Can we find it?"

7. Kevin has now figured out the answers to these two examples. He is now relieved. He says "Let's consider my dad's situation now. Over the last five years, my dad had annual returns of 21%, 7%, -19%, 16%, and 13%. I wonder what his average annual return was with the two methods (i.e. arithmetic and geometric returns)."

8. Do you think his dad's arithmetic average return and geometric average return were similar? Why or why not?

9. Do you think is one of these measures (i.e. arithmetic or geometric average return) always better than the other one?

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