Integrated Planning and Scheduling in Transportation Infrastructure Maintenance Management

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1 ABSTRACT

Transportation infrastructure systems are vital to the welfare of modern society. The management of maintenance and rehabilitation activities of the transportation infrastructure systems mainly focus on two types of problems: 1) the planning problem, in which an agency with a limited budget identifies suitable maintenance treatments for the facilities and decide at which year the activities should be performed, and 2) the scheduling problem, in which the agency develops a schedule to incorporate the time at which the maintenance treatments begin and end based on the availability of daily resources. Existing studies mainly focus on the maintenance planning problem of selecting the maintenance treatment for the overall benefit of the system without considering the scheduling of daily works. However, without the proper scheduling of maintenance activities, these approaches are far less effective and could lead to overplanning of maintenance activities and shortage of resources. Motivated by these facts, this paper proposes to address the integration of planning and scheduling of transportation infrastructure maintenance activities. In this paper, a mixed integer linear programming formulation was developed to help the planning and scheduling of maintenance activities and a heuristic algorithm was proposed to solve the problem efficiently. Example networks were tested for the performance comparison between the CPLEX solver and the heuristic algorithm. The results show that the proposed model can help transportation agencies better manage their maintenance and rehabilitation activities.

Keywords: Infrastructure Management, Asset Management, Pavement Management, Maintenance Planning, Project Scheduling

2 INTRODUCTION

Infrastructure is the spine of the economy and a fundamental input to every economic output. It is crucial to the nation's success and the public health and welfare. Once every four years, a Report Card for America's infrastructure is published by the American Society of Civil Engineers (ASCE), which grades the current state of the national infrastructure categories on a scale of A through F. Ever since the year 1998, America's infrastructure has obtained constant D averages, and failure to

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close the investment gap with required improvements and maintenance has continued. Facilities in poor condition lead to escalation in operating costs for cars, trucks, and rail vehicles. Additional costs include damage to vehicles from imposition of both additional miles traveled, deteriorated roadway surfaces, time used to avoid unusable or densely congested roadways or because of breakdown of transit vehicles, and the additional cost of repairing facilities after they have deteriorated as opposed to preserving the facilities in good condition. In addition, increased congestion reduces the reliability of the transportation facilities, meaning that travelers allot more time for trips to guarantee on-time arrivals (and for freight vehicles, on-time delivery). Moreover, increased congestion increases the environmental and safety costs by forcing more travelers to substandard travel conditions and vehicles to operate at less efficient levels as the conditions continue to deteriorate.

In this light, proper maintenance of transportation infrastructure systems is of vital importance, due to the large capital expenditure and construction time needed to construct new facilities. At the same time, maintenance activities have nontrivial costs, and agencies responsible for maintenance have limited budgets, making it necessary to determine what maintenance actions to perform, and when, in order to ensure a well-functioning system with a reasonable cost. Broadly speaking, two separate problems need to be solved: the planning problem, in which an agency with an allotted budget must identify suitable maintenance treatments for the facilities which have to undergo maintenance and decide at which time the activities must be performed, and the scheduling problem, in which the agency needs to develop a schedule to incorporate the time at which the maintenance treatments begin on the facility and when they end based on the availability of resources.

3 RELATED WORKS

Many existing studies on network-level transportation infrastructure maintenance planning problems developed models based on Markov decision processes (MDP). In a MDP, deterioration is represented by transition probabilities, which can be determined based on expert judgment or empirical observations. To obtain optimal maintenance policies from the MDP models, previous studies usually used dynamic programming (DP) [1,2,3,4] or linear programming (LP) [2,5,6,7,8].

Other researchers used mixed-integer linear/nonlinear programming formulation [9,10,11,12]. Genetic algorithm and other evolutionary models have also been used to find optimal maintenance plan [13,14,15,16]. For example, Maji and Jha [17] developed a nonlinear nonconvex formulation for the maintenance optimization problem in highway infrastructure maintenance schedule with budget constraint. A genetic algorithm is utilized for the solution procedure. Marcous and Lounis [18] applied genetic algorithms to maintenance optimization because of their robust search capabilities that resolve the computational complexity of large-size optimization problems. In the proposed approach, Markov-chain models are used for predicting the performance of infrastructure facilities because of their ability to capture the time-dependence and uncertainty of the deterioration process, maintenance operations, and initial condition, as well as their practicality for network level analysis.

Stochastic programming has also been applied to address the uncertainty in transportation infrastructure maintenance management. For example, Li and Puyan [19] developed a stochastic optimization model for project selection that considers budget uncertainty. The model was formulated as a stochastic multi-choice multidimensional knapsack problem with budget recourses. Gao et al. [20]. propose a multistage, stochastic programming model to adopt maintenance and rehabilitation scheduling policies that consider all budget fluctuations. The authors develop a solution procedure using the augmented Lagrangian decomposition algorithm.

Despite there being many studies on infrastructure maintenance planning problems, there are few studies related to the scheduling of maintenance activities. Fwa et al. [21] developed a mathematical programming procedure for routine maintenance activities at the network level developed for incorporation into the existing pavement maintenance-management system of Indiana. The procedure allowed a highway agency to determine amounts of different routine maintenance activity types to be performed over a given time period under the constraints of production requirements, budget allocation, manpower, material and equipment availability, and pavement rehabilitation schedule. But the deterioration of pavement was not considered in this paper.

As discussed above, existing studies focus on either the planning of maintenance activities over certain time periods or the scheduling of daily maintenance work. But no studies has been done on integrating the planning and scheduling problem together. The consequence of this lack of research is that there may be an excessive planning of maintenance activities and not enough daily resources available to schedule the maintenance work over a given time period. Therefore, in this paper, we propose developing an integrated model that takes both planning and scheduling of maintenance activities into consideration.

4 METHODOLOGY

The formulation of the proposed model is discussed in this section. First, the planning problem and scheduling problem are discussed separately. Then, the proposed integrated model is presented. The sets, parameters, and variables mentioned in the model description are summarized in Table 1.

Table 1: Notations

Term	Definition
Sets	
I	Set of infrastructure facilities
M	Set of maintenance treatments
T	Set of maintenance planning periods
K	Set of resources required to implement maintenance
H	Set of scheduling horizon
Parameters	
B_t	Budget available for maintenance in the tth time period, $t \in T$
c_m	The cost of applying the mth treatment, $m \in M$
e_m	The effectiveness of the mth treatment, $m \in M$
ρ	Annual deterioration rate of facilities
g_{mk}	The kth resource needed for the mth maintenance, $k \in K$, $m \in M$
p_m	Duration of maintenance treatment for the mth facility, $m \in M$
G_k	The kth resource available for each work day
m_i	Selected maintenance treatment for the ith facility after solving the planning problem (1)-(7), $m_i \in M, i \in I$
Variables	
x_{it}	Condition of the ith facility at the end of the tth time period, $i \in I, t \in \{0\} \cup T$
s_i	Initial condition of the ith facility, $i \in I$
z_{id}	Binary variable indicating that the maintenance activity of the ith facility will start on day d, $i \in I$, $d \in H$
y_d	Binary variable indicting all activities are completed on day d, $d \in H$
y_{itm}	Binary variable indicating whether the mth maintenance treatment is applied to the ith facility in the tth time period, if it is, $y_{itm} = 1$, otherwise $y_{itm} = 0$. $i \in I$, $m \in M$, $t \in T$.
u_{itmd}	Binary variable and if $u_{itmd} = 1$, the mth maintenance treatment will be applied to the ith facility starting on day d in year t. $i \in I$, $m \in M$, $t \in T$,$d \in H$.

4.1 Infrastructure Maintenance Planning Problem

A typical infrastructure maintenance planning problem (1)-(7) is to find the optimal maintenance plan so that the network-level condition is maximized. Maintenance treatments are assumed to be carried out at the end of each year. Decision-makers have to decide annually which facility should be maintained, when it should maintain and which treatment should be implemented at the facility. The maintenance works are subject to yearly budget constraints.

$$\max \frac{1}{|I| \times |T|} \sum_{i \in I} \sum_{t \in T} x_{it} \tag{1}$$

$$\text{subject to: } x_{i0} = s_i, \forall i \in I \tag{2}$$

$$x_{it} = \rho x_{i,t-1} + \sum_{m \in M} e_m y_{itm}, \forall i \in I, \forall t \in T \tag{3}$$

$$\sum_{m\in M} y_{itm} \leq 1, \forall i \in I, \forall t \in T \tag{4}$$

$$\sum_{i\in I}\sum_{m\in M} c_m y_{itm} \leq B_t, \forall t \in T \tag{5}$$

$$0 \leq x_{it} \leq 100, \forall i \in I, \forall t \in T \tag{6}$$

$$y_{itm} \in \{0,1\}, \forall i \in I, \forall t \in T, \forall m \in M \tag{7}$$

The objective function (1) maximizes the average condition of all facilities over all planning horizons. $|\cdot|$ represents the cardinality of a set. Constraint (2) assigns (known) initial conditions to each facility. Constraint (3) states that the condition of the ith facility at the end of tth year is determined by its condition at previous year multiplied by a deterioration rate and the effectiveness of the applied maintenance treatment. Constraint (4) states that each facility cannot receive more than one maintenance treatment in a single year. Constraint (5) gives a budget limitation on the annual maintenance expenditures. Constraints (6) and (7) define the decision variables x_{it} and y_{itm}.

4.2 Resource Constrained Scheduling Problem

When the maintenance activities are determined for each facility through (1)-(7), a resource-constrained project scheduling problem (8)-(13) can be used to find a schedule of minimal duration by assigning a start time to each activity and also taking the resource availabilities into consideration.

$$\min \sum_{d\in H} d y_d \tag{8}$$

$$\text{subject to: } \sum_{d\in H} d y_d \geq \sum_{d\in H} d z_{id} + p_i, \forall i \in I \tag{9}$$

$$\sum_{d\in H} z_{id} = 1, \forall i \in I \tag{10}$$

$$\sum_{i\in I} g_{m_i k} \sum_{\tau=d-p_i+1}^{d} z_{i\tau} \leq G_k, \forall k \in K, \forall d \in H \tag{11}$$

$$y_d \in \{0,1\}, \forall d \in H \tag{12}$$

$$z_{id} \in \{0,1\}, i \in I, d \in H \tag{13}$$

The objective function (8) minimizes the number of days to complete all activities over the scheduling horizon. Constraint (9) defines the end day of all activity. Constraint (10) ensures each activity to be scheduled has only one start day. Constraint (11) checks if each resource is available for the duration of the activity.

4.3 Integrated Maintenance and Scheduling Problem

In the proposed integrated model (14)-(23), it is assumed that at the end of each year maintenance treatments are performed. It is the responsibility of the decision makers to decide annually based on the initial condition of the facility, the available resources and budget the facility to be maintained, the maintenance treatment to be implemented and the time at which these should be done on the facility. The objective function (14) of the proposed model is to maximize the average condition of the infrastructure systems over the planning horizon.

$$\max \frac{1}{\mid I \mid \times \mid T \mid} \sum_{i \in I} \sum_{t \in T} x_{it} \tag{14}$$

Equations (15)-(16) show that the condition of the ith facility at the tth year is determined by the initial condition x_{io}, deterioration rate ρ and maintenance effectiveness $e_m$over the years.

$$x_{i0} = s_i, \forall i \in I \tag{15}$$

$$x_{it} = \rho x_{i,t-1} + \sum_{m \in M} e_m y_{itm}, \forall i \in I, \forall t \in T \tag{16}$$

Equation (17) limits the number of funded treatments per year for a specific facility.

$$\sum_{m \in M} y_{itm} \leq 1, \forall i \in I, t \in T \tag{17}$$

Equation (18) is the budget constraint, which restricts the maintenance expenditure to be below a given budget, where c_m is the cost for the mth treatment and B_t is the available budget in year t.

$$\sum_{i \in I} \sum_{m \in M} c_m y_{itm} \leq B_t, \forall t \in T \tag{18}$$

Equation (19) ensures that the variable u_{itmd} equals to 0 if no maintenance treatment is selected for the ith facility in year t. It also enforces that there is only one starting day if a maintenance treatment is selected.

$$\sum_{d \in H} u_{itmd} = y_{itm}, \forall i \in I, t \in T, m \in M \tag{19}$$

Constraint (20) considers the available daily resources G_k, and checks if there is sufficient resources available for the duration of each planned maintenance activity.

$$\sum_{i \in I} \sum_{m \in M} g_{mk} \sum_{\tau = d - p_m + 1}^{d} u_{itm\tau} \leq G_k, \forall d \in H, k \in K, t \in T \tag{20}$$

Constraint (21) restricts the condition of the facility between 0 to 100.

$$0 \leq x_{it} \leq 100, \forall i \in I, t \in T \tag{21}$$

Constraint (22) defines decision variables y_{itm} which is a binary variable indicating whether the mth maintenance treatment is applied to the ith facility in the tth time period.

$$y_{itm} \in \{0,1\}, \forall i \in I, t \in T, m \in M \tag{22}$$

Finally, constraint (23) defines decision variable u_{itmd} which is a binary variable indicating the day d of when the maintenance activity will begin, of the ith facility receiving the mth treatment at year t.

$$u_{itmd} \in \{0,1\}, \forall i \in I, t \in T, m \in M, d \in H \tag{23}$$

4.4 Heuristic Algorithm

One limitation of the above model formulation is that the size of the problem grows exponentially and therefore incurs prohibitive computational time when the number of facilities increases. To circumvent this problem, we developed a three-phase heuristic algorithm to solve the proposed model.

In the first phase, the maintenance treatment with the highest cost-benefit ratio is selected for each facility so that it will stay in good condition for the upcoming year. The selection is determined by considering the deterioration of the facility and the effectiveness of the maintenance treatment.

In the second phase, the budget is allocated to the facilities having highest cost benefit ratio. The remaining budget is allocated to the remaining facilities in the same manner. This process is repeated until the budget is exhausted. The facilities having the worst condition are preferred in cases where multiple facilities have the same maintenance level of need.

In the third phase, the selected activities is scheduled. The algorithm checks each day of the planning horizon starting from the first day. If the resources are available for the whole duration of the maintenance activity, then the activity begins on that day. Otherwise, the algorithm will search the next day and so on. These three phases will be repeated for the planning horizon.

5 CASE STUDY

In this case study, two road network examples are presented to illustrate the proposed integrated planning and scheduling maintenance problem and the developed algorithm. One is a small size problem and the other is a large size problem. While the exact solution of the first example is obtained using the CPLEX solver, the second example is solved using heuristic algorithm.

5.1 Example 1

For illustration purposes, this example maintenance planning problem has 50 pavement sections. The planning horizon is assumed to be 4 years. During the planning horizon, all road sections are eligible for maintenance treatments, which are assumed to be applied at the end of each year. The annual budget is set at $1,000,000. The condition score (CS) is selected as the pavement condition indicator. Condition score represents the pavement's overall condition in terms of both distress and ride quality (serviceability index values). It ranges from 1 (the worst condition) to 100 (the best condition) [22].

For demonstration purposes, the deterioration rate ρ is set at 0.95. The selection of the deterioration rate is taken from previous studies (e.g., [9]). The daily resource available is considered to be 30 manpower and 20 machinary. As shown in Table 2, five maintenance treatments options were used in this case study. The maintenance treatments cost c_m, effectiveness e_m, the typical maintenance treatments, daily resource and duration were prepared on the basis of information from previous studies [7, 9, 21, 23, 24, 25].

Table 2: Cost and Effectiveness of Maintenance Treatments

Notations in proposed model (m)	Maintenance treatment	Maintenance treatment unit cost ($1000)	Average condition score increase	Daily resource required (manpower, machinary)	Duration (Days)
	Needs Nothing (NN)	0	0	0	0
1	Preventive maintenance (PM) includes Seal Coats (Chip Seals), Thin Overlays < 2”, and Micro-Surfacing	6.1	3	(6,2)	2
2	Light rehabilitation (LRhb) includes 2” ≤ Overlays < 3”, Widening Pavement and Seal Coat, Base Repairs and Seal coat, Mill, Seal, and Thin Overlay	21	15	(9,3)	7
3	Medium rehabilitation (MRhb) includes 3” ≤ Overlays < 5”, Mill and Inlay (Mill and Fill), Mill, Stabilize Base, and Seal, Level Up and Overlay, Base Repairs and Overlay	46	25	(9,5)	21
4	Heavy rehabilitation (HRhb) includes Full Pavement Reconstruction, Bomag, Add Base, and Overlay or Seal	110	40	(11,7)	90

The first year optimal solution is shown in Figure 1. The planned maintenance activities are scheduled over the duration of a calendar year with 261 work days. The schedule shows the selected maintenance treatment of different sections, the start day of each treatment, and the duration of each treatment. For example, section 36 will be maintained with medium rehabilitation, which takes 21 days to finish. The maintenance project will start on day 132 and end on day 153.

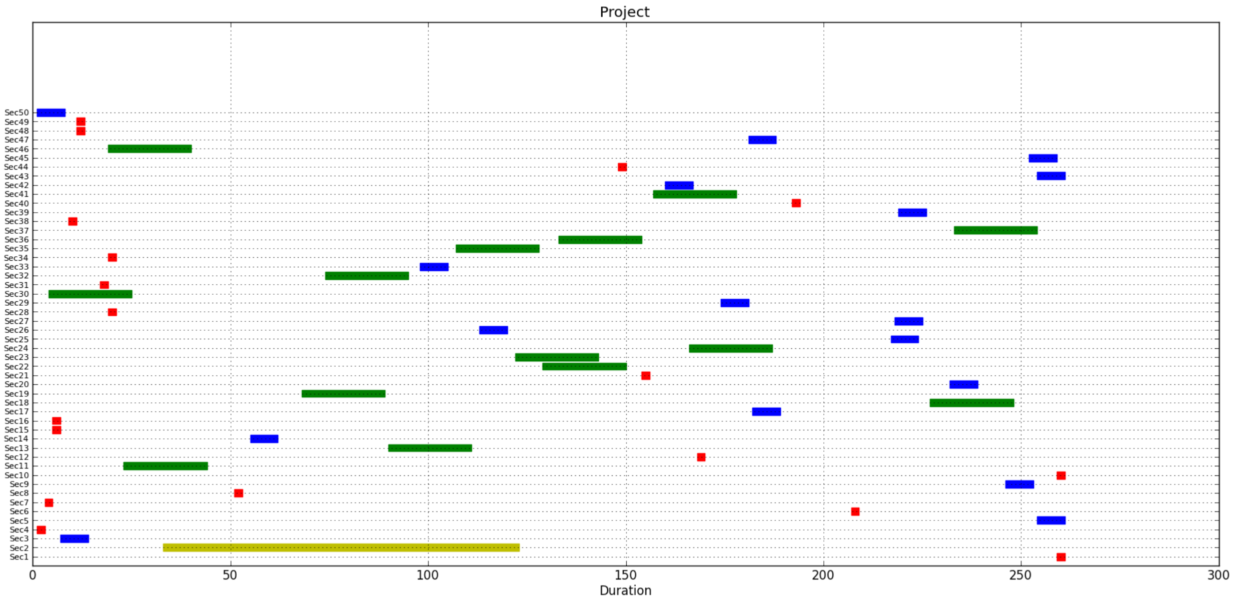
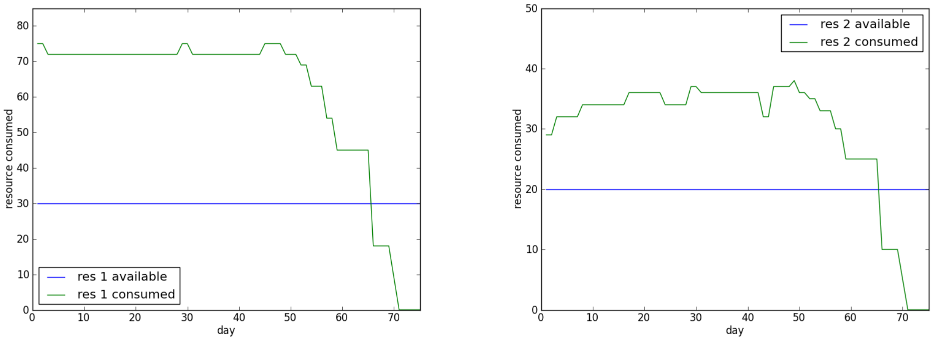


Figure 1: Maintenance Activity Schedule of First Year (Red = Preventive maintenance, Blue = Light rehabilitation, Green = Medium rehabilition, Yellow = Heavy rehabilitation)

To check whether the resources are being utilised efficiently, the resource consumption is calculated in two scenarios: 1). the planning problem and the scheduling problem of maintenance activities are solved separately, and 2). the proposed integrated model is solved.

Figure 2 shows the first scenario. As can be seen in Figure 2, to complete the maintenance activities on the selected facilities, the daily resources required are more than double the available daily resources. Since only planning is considered, the amount of maintenance activities selected are higher and the consumption of resource 1 and 2 is higher than the available daily resources.

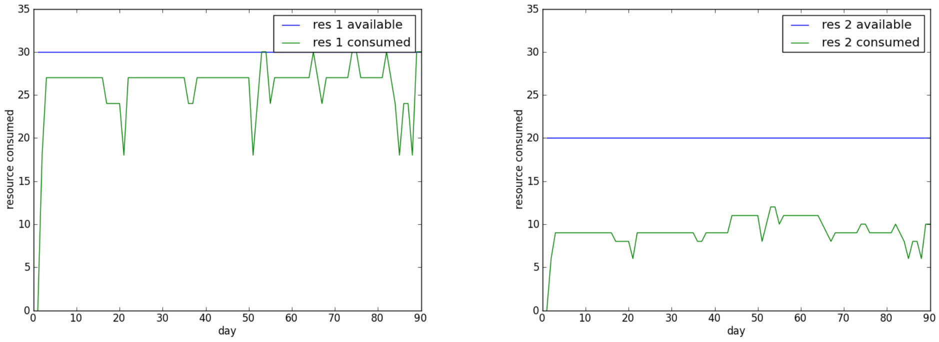


(a) Consumption of resource 1

(b) Consumption of resource 2

Figure 2: Consumption of resources when only planning is considered.

When the integrated planning and scheduling problem is solved, the resource consumption is shown in Figure 3. As can be seen in Figure 3, the maintenance activities of the selected facilities are scheduled in an optimal manner and the daily resource consumption of resource 1 and 2 are within the available daily resources since both planning and scheduling are considered.



(a) Consumption of resource 1 (b) Consumption of resource 2

Figure 3: Consumption of resources when only planning and scheduling is considered.

5.2 Example 2

In Example 2, the integrated maintenance planning and scheduling problem was solved for a road network with 1,000 pavement sections using the heuristic algorithm. The purpose of this example is to test the computational efficiency of the proposed heuristic algorithm when it is applied to practical-sized problems. The choices of maintenance treatments and deterioration rate are assumed the same as in example 1.

Figure 4 shows the schedule of the sections as per the maintenance treatment selected. Out of the 1,000 pavement sections, 87 are scheduled maintenance in the first year because of limited budget and resources. This schedule shows the type of maintenance treatment selected for the pavement section, the start day of the treatment and the day the duration of the maintenance treatment. Further experiment shows that the heuristic algorithm is able to solve the integrated planning and scheduling problem with up to 10,000 sections in 2 minutes and 23 seconds.

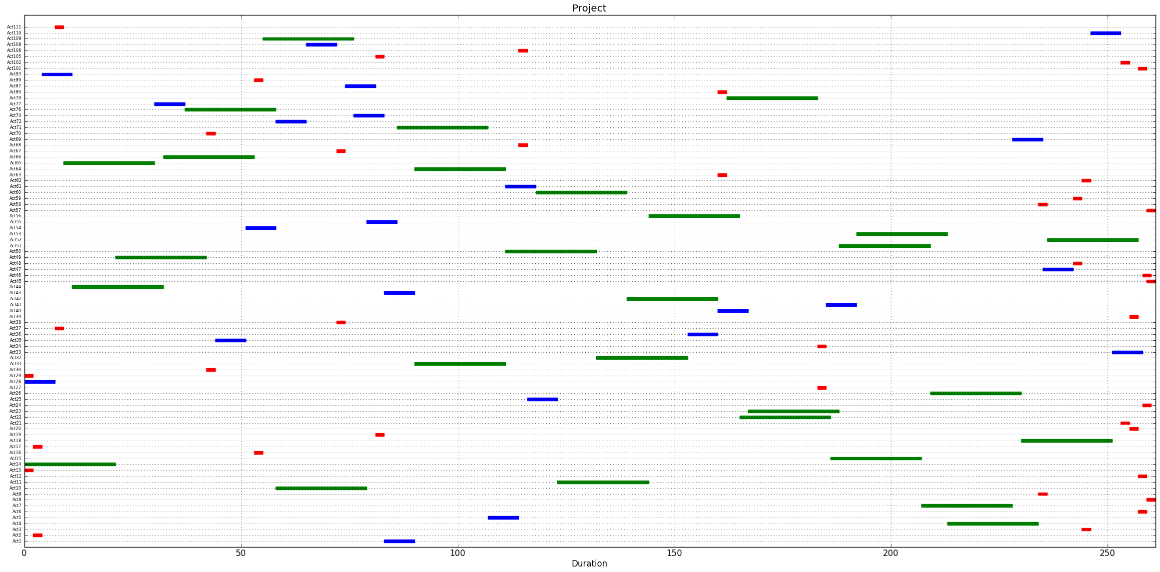


Figure 4: Maintenance Activity Schedule of First Year(Red = Preventive maintenance, Blue = Light rehabilitation, Green = Medium rehabilitation, Yellow = Heavy rehabilitation)

6 CONCLUSION

In this paper, the authors developed an integer linear programming model that integrates both maintenance planning and scheduling problems for transportation infrastructure maintenance management. In the developed model, the authors take into consideration of the maintenance type, cost and effectiveness. The daily resources (both manpower and equipment) available is also included in the model for the scheduling of maintenance activities.

The authors also developed a three-phase heuristic algorithm to solve the proposed model, that produces near-optimal solutions with computational time being reduced substantially as compared to the CPLEX solver. Thanks to this substantial time saving, computationally efficient solutions to more complicated network-level problems can be formulated by using heuristic as a building block.

Two road network examples are presented to illustrate the proposed integrated planning and scheduling maintenance problem and the developed algorithm. The case study results show that the developed heuristic algorithm give good results compared with the results from the CPLEX solver. The two examples also illustrate the daily consumption of resources for two scenarios. As a result, the resource consumption when considering both the planning and scheduling together provides a more feasible and optimal solution.

In this research, the developed model has only considered an in-house approach, in which the infrastructure management agency uses the resources and equipment that are available within the agency. For future research, the model can also include the scenarios where the maintenance work can be outsourced to contractors if the planned work cannot be completed by internal resources.

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