Forecasting Value-at-Risk of Asian Stock Markets Using the RDCC-GARCH Model Under Different Distributional Assumptions

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Abstract. The aim of this paper was to accurately and efficiently forecast from multivariate generalized autoregressive conditional heteroscedastic models. The Rotated Dynamic Conditional Correlation (RDCC) model with the Normal, Student's-*t* and Multivariate Exponential Power distributions for errors were used to account for heavy tails commonly observed in financial time series data. The daily stock price data of Karachi, Bombay, Kuala Lumpur and Singapore stock exchanges from January 2008 to December 2017 were used. The predictive capability of RDCC models, with various error distributions, in forecasting one-day-ahead Value-at-Risk (VaR) was assessed by several back-testing procedures. The empirical results of the study revealed that the RDCC model with Student's-t distribution produced more accurate and reliable risk forecasts than other competing models.

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1. Introduction:

Financial markets are considered to have an intense role in economic conditions for countries all over the world. In this regard, one of the most important characteristics of financial markets is to model and estimate the volatility. Modeling volatility is important as it displays the dynamic fluctuations in stock prices (Raja and Selvam, 2011). A measure of uncertainty for changes in asset prices is considered as the volatility and it was used earlier by Markowitz (1952) as a measure of risk.

It is a well-established fact that volatility varies over a specific period of time and tends to cluster in periods: large changes in stock prices tend to have colossal changes in prices, and minor changes in stock prices tend to have small changes in prices. This phenomenon, when standard deviation differs over time, is called heteroscedasticity. Furthermore, the volatility is found to be autocorrelated, which elaborates that today's volatility depends upon the former volatilities.

Measuring and estimating the stock price volatility is a significant perception in finance in wide-ranging, and an investment decision, owing to its dynamic behavior. That headed the researchers to anticipate various statistical and mathematical models to capture the volatility of stock earnings in financial markets all over the world. The innovative study in this field was put forward by Engle (1982), who first proposed the autoregressive conditional heteroscedastic (ARCH) model that permits the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Bollerslev (1986) suggested the generalized ARCH (GARCH) model permits longer memory and a more flexible lag structure. In this model, the conditional variance is modeled as a linear function of past sample variance, on the other hand, it also allows lagged conditional variance to enter in the model.

As financial volatilities differ with the passage of time, so it is crucial to predict the dependence in the co-movements of returns. Bollerslev et al. (1988) introduced a multivariate GARCH (MGARCH) model to estimate the volatilities and covolatilities for more than one asset returns. They defined the MGARCH model such as all lagged conditional variances and covariance were functions of every conditional variance and covariance as a cross product of squared and lagged square returns and observed that the MGARCH model brings flexibility in multivariate modeling. The number of estimated parameters increases with the increase in the dimension of the system in MGARCH models and to overcome this problem Bollerslev (1990) presented the Constant Conditional Correlation GARCH (CCC-GARCH) model. In the CCC-GARCH model, the conditional correlations are constant, and the conditional correlation is proportional to the product of the corresponding conditional standard deviation. Bauwens et al. (2006) provided a comprehensive review of multivariate GARCH models.

Although the CCC-GARCH model was computationally attractive but the assumption of constant conditional correlation was found to be too restrictive. Tse and Tsui (2002) introduced a Time-Varying Correlation (TVC) model to capture the time-varying correlations of assets over time. Engle (2002) introduced the Dynamic Conditional Correlation GARCH (DCC-GARCH) model that could measure time-varying correlations and forecast the future correlations in large dimensions. Tsay (2006) reviewed the multivariate volatility models and concluded that a simple DCC model estimates the conditional variances and correlations jointly and satisfies the positive definite constraints.

The Dynamic Conditional Correlation DCC model of variances and correlations with economic loss function was evaluated by Engle and Colacito (2006). They constructed portfolios to reduce the predicted variance of returns and estimates the accuracy of the covariance matrix by using asymmetric DCC, DCC-Mean Reverting (DCC-MR) and Baba, Engle, Kroner, and Kraft (BEKK) models and concluded that results acquired from asymmetric DCC models were found to be significant. Cappiello et al. (2006) proposed the Asymmetric Generalized DCC (AG-DCC) model that was applicable to study correlation dynamics among various asset returns and investigated the existence of asymmetric responses in conditional variances and correlations.

Iqbal (2013a, 2013b) considered a robust estimation of variants of multivariate GARCH models. To study the financial data in high dimensions, Noureldin et al. (2014) proposed a new class of multivariate model labeled as Rotated Autoregressive Conditional Heteroscedastic (RARCH) model. They showed a way to fit multivariate GARCH models with the help of targeting covariance. The primary structure was to rotate the returns on various BEKK and DCC models called them the rotated DCC (RDCC) and rotated BEKK (RBEKK) models. They applied various dynamic specifications of RBEKK and RDCC models and showed that these models provide better results as compared to OGARCH and GOGARCH models.

Braione and Scholtes (2016) used the RBEKK model and showed that for an accurate forecast of Value-at-Risk (VaR) the fat tailedness and skewness must be considered for better VaR forecast. The Normal, Student's-*t* and Multivariate Exponential Power distributions and their skewed versions were used for VaR forecasting and the accuracy assessed using backtesting procedures and concluded that skew-Student's-*t* outperforms other models. Nieto and Ruiz (2016) focused on VaR forecast, under practical applications of insurance and financial institutions and evaluate the time series of financial returns.

The focus of this study was to forecast one-day-ahead VaR from RDCC GARCH models with Normal and heavy-tailed distributions such as Student's-*t* and Multivariate Exponential Power distributions. The daily stock exchange data of four Asian stock markets (Pakistan, India, Malaysia, and Singapore) were used. The RDCC model was used to estimate and forecast the VaR and various backtesting procedures were applied to quantify the accuracy of VaR forecast at 1% and 5% confidence levels. For the growth of the economy, better models of volatility and accurate forecasts of risk are necessary. The findings of this study may help in providing accurate and reliable forecasts for high dimension asset returns.

2. The Rotated DCC-GARCH Model:

In this research, we applied the efficient multivariate GARCH model called the Rotated Dynamics Conditional Correlation (RDCC) model proposed by Noureldin et al. (2014). This model works on the transformation of devolatized returns and enables target variances and covariances approach. This model enables one to estimate and forecast financial data with many assets and parameters.

The returns for the individual series are calculated on the log difference of the prices of the series.

$$r_{i,t} = 100\left[ln\left(P_{i,t}\right) - ln\left(P_{i,t-1}\right)\right]$$

In multivariate formation, vector $\boldsymbol{\varepsilon_t}$ depend on the set of information I_t with zero mean and returns variance H_t. In general, we can express as

$$(\boldsymbol{\varepsilon_t}\,|I_t) \sim N(0,\, H_t)$$

In RDCC models r_{it} are the returns for $t = 1,2,3,\cdots,T,$ with $i = 1,2,\ldots N$ dimensional vector of asset returns. The mean return, with a given set of information, is zero, i.e., $E\left(r_{i,t}\middle|I_{t-1}\right) = 0$ and the conditional covariance $Var\left(r_{i,t}\middle|I_{t-1}\right) = H_t$. The standard DCC model of Engle (2002) decomposed the variance-covariance matrix as,

$$H_t = D_t R_t D_t$$

where R_t is the conditional correlation matrix of r_{it} and D_t is a diagonal matrix of conditional standard deviations, i.e. $D_t = diag\left(\sqrt{h_{ii,t}}\right)$ for $i = 1,2,\cdots,N.$ Conditional variances $h_{ii,t}$ are typically described by GARCH-type models. The standardized returns of potentially correlated returns are $\varepsilon_t = D_t^{-1} r_t$, where $r_t = \left(r_{1,t}, r_{2,t}, \cdots, r_{N,t}\right)$ is the vector of daily returns.

The unconditional covariances of returns are $\Pi = var\langle\varepsilon_t\rangle$, and the conditional correlation is expressed by the following relationship:

$$R_t = (Q_t \otimes I_N)^{1/2} Q_t (Q_t \otimes I_N)^{-1/2} \quad (2.1)$$

where $\otimes$ is element-wise Hadamard product, Q_t is $N \times N$ covariance matrix of ε_t and I_N is the N-dimensional identity matrix.

The RDCC models is defined as:

$$Q_t = (1 - \alpha - \beta)\Pi + \alpha \varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} \quad (2.2)$$

Here we assume $\alpha > 0$ and $\beta \geq 0$ and covariance stationary requires $\alpha + \beta < 1$. This confirms that R_t is the correlation matrix. For multiple assets where volatility estimation prediction and forecast are more flexible, we can write as:

$$Q_t = (\Pi - A\Pi A' - B\Pi B') + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B' \quad (2.3)$$

Ensuring the condition of positive semi-definite matrix $(\Pi - A\Pi A' - B\Pi B')$,

It is clear from Equation (2.3) that rotation of standardizing returns enables us to fit into the specification of the flexible dynamic of RDCC, Non-standardized returns are decomposed as

$$\Pi = P_c \Lambda_c P'_c$$

P_c is the eigenvector and Λ_c holds the eigenvalues on the main diagonal.

The rotated standardized returns are constructed as

$$\tilde{\varepsilon}_t = P_c \Lambda_c^{-1/2} P'_c \varepsilon_t$$

where $var(\tilde{\varepsilon}_t) = I_N$ so, the variances of rotated standardized returns highlighted by

$$Q_t^* = \text{var}\langle \tilde{\varepsilon}_t | I_{t-1} \rangle$$

$$Q_t^* = (I_N - AA' - BB') + A\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}A' + BQ_{t-1}^*B' \quad (2.4)$$

$Q_t^* = I_N$ For $t = 0$

The correlation matrix is constructed as

$$Q_t = P_c \Lambda_c^{-1/2} P'_c \, Q_t^* P_c \Lambda_c^{-1/2} P'_c$$

The correlation matrix constructed according to Equation (2.1)

Setting $A = \alpha^{1/2} I_N$ and $B = \beta^{1/2} I_N$ gives Scalar-RDCC model.

This type of specification controls all elements of Q_t^T to share the common dynamics of the parameters. The process is co-variance stationary under the assumption of $\alpha + \beta < 1$.

The terms providing scalar RDCC model $\varepsilon_t = D_t^{-1} r_t$

Are standardized correlated returns.

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B') + A\varepsilon_{t-1}^T\varepsilon_{t-1}A' + BQ_{t-1}B' \quad (2.5)$$

where A and B are $N \times N$ parameter matrices. $\varepsilon_t = D_t^{-\frac{1}{2}} r_t$ are the standardized returns and $\bar{Q} = var\langle \varepsilon_t \rangle$ is the unconditional matrix of ε_t: The term $\bar{Q} - A\,\bar{Q}\,A' - B\,\bar{Q}B'$ must be positive semi-definite matrix.

The defined model is the DCC parameterization of targeted correlations, which means Q_t revert to $\bar{Q}$ in the covariance stationary model. The DCC targeted-correlation model estimated by Maximum Likelihood Estimation (ML-estimation) because, Q_t is estimated separately in the first stage and method of the moment is implemented to estimate $\bar{Q}$. A class of distributions for standardized error $\tilde{\varepsilon}_t$, such as multivariate Normal, Student's-t and Multivariate Exponential Power distributions can be used, and the resulting models are called RDCC-N, RDCC-T, RDCC-MEP models, respectively.

3. Value-at-Risk:

In financial assets returns volatility measures how much prices move each day (a weak, a month or a year). High volatility means higher profit or risk so volatility is a measure of risk and Value-at-Risk (VaR) is a measure of market risk α% maximum loss with the specified time horizon. In this section, we forecast maximum loss (VaR) to RDCC models for portfolios. We are interested that the returns of the portfolio's fall below a certain limit; that certain limit is VaR. It measures the market risk of the portfolios, generated with the degree of confidence over the time horizon. To calculate VaR for portfolio returns, we manipulate the quantile as a relative term, at significance level α.

3.1. VaR Estimation

Let r_t indicated as N-dimensional time vector of daily returns at time t and w_t is the vector of equal weights up to time $t - 1$. The portfolio returns are obtained as

$$r_{p,t} = w'_{t-1} r_t$$

and the portfolio VaR at time t:

$$VaR_{t,\alpha} = \sigma_{p,t} q_\alpha, \quad (3.3)$$

$$\sigma_{p,t}^2 = w'_{t-1} H_{t|t-1} w_{t-1}$$

$H_{t|t-1}$: Conditional covariance matrix of returns under given information set at time $t - 1$,

w_t: N-dimensional vector of equal weights, N denotes the number of assets,

$$w_t = (w_{1,t}, \cdots, w_{N,t}), \; w_i = \frac{1}{N}$$

$\sigma_{p,t}$: is the portfolio standard deviation and q_α is the left quantile of the conditional distribution at α%. To obtain a one-step-ahead forecast of conditional variance-covariance matrix recursively as:

$$\hat{H}_{t+1} = E(H_{t+1} | I_t) \quad (3.4)$$

To do h-step ahead forecast:

$$\hat{H}_{t+h} = E(H_{t+h}|I_t) \quad (3.5)$$

To obtain portfolio VaR forecasts we use the notation:

$$VaR_{t+h,\alpha} = \hat{\sigma}_{p,t+h} q_\alpha \quad (3.6)$$

Several methods are available for the calculation of VaR but we used two different approaches, the parametric approach and the Non-Parametric approach. The Non-Parametric approach comprises historical simulation which is based on historical data. Whereas the parametric method makes an assumption on the distribution of the returns. The parametric approach can be alienated into two parts (Dowd, 2002):

The conditional parametric approach is based on GARCH type models the volatility can be modeled with any GARCH family model (Univariate or Multivariate), and then the fitted model is used for the estimation of VaR.

In an unconditional parametric approach, the volatility of the returns is time-invariant, and they do not depend upon the period at which returns are observed, the calculated volatility based on all the returns is then used for the estimation of VaR.

3.2. Historical Simulation

Historical simulation (HS) is a method of calculating VaR, it uses past data of the returns to calculate VaR forecasts. HS forecast forthcoming losses based on historical performance. In the multivariate case at first, the vectors of the weights for the portfolio need to be defined and the sum of elements must be equals to unity. The HS in the multivariate case can be calculated in the following steps (Danielsson, 2011):

Compute sample portfolio returns, $r_{p,t} = w'_{t-1} r_t$.

Assort the sample portfolio returns and denote it by $\{y_{port}\}$.

Calculate $\alpha\%$ quantile of the sample portfolio returns, $q = \alpha * T$.

Extract q^{th} value from the assorted returns $\{y_{port}\}$.

Historical simulation VaR, $VaR_{port} = y_{port}[\alpha * T]$.

3.3. Unconditional Parametric Approach

Parametric methods used for the estimation of VaR are based on the distribution of the returns. The parametric approach evaluates VaR directly from the volatility of the returns. In the unconditional parametric approach, the volatility which is obtained from the distribution of returns remains constant throughout the VaR period (Danielsson 2011). In this research, we derived VaR with Normal, Student's-*t* and multivariate Exponential Power distributions.

4. Backtesting:

Wald (1950) introduced the loss function for measuring the difference between actual and estimated values of the data. A loss function is an approach applied to rank the accuracy of forecasted VaR by (Piontek, 2014). Romero et al. (2014) indicated that various backtesting procedures were applied in the literature, classified into two groups: backtesting based on loss function and backtesting based on statistical tests. In this research, we apply backtesting for tests and as well as for loss function. Backtesting is a check on the accuracy of VaR forecasts and assesses the relationship of potential loss and accuracy of risk forecast. To predict the accuracy of forecasted VaR, we implement multiple methods regarding statistical backtesting criteria and apply those tests we take start from indicator function:

$$I_t(\alpha) = \begin{cases} 1, & if\ r_{p,t} \leq VaR(\alpha) \\ 0, & if\ r_{p,t} \geq VaR(\alpha) \end{cases} \quad (4.1)$$

where I_t is the indicator function with value 1 if the predicted VaR exceed than the portfolio returns at the time $(t-1)$ according to Christoffersen (1998), elsewhere I_t is zero. The sequence must satisfy the independence of exception and two properties of the correct failure rate, and then $\alpha * 100\%$ probability is the probability of VaR violation. These properties combined into one statement to assess that, this hit function independently identically distributed (iid) with Bernoulli random variable by the probability P, $I_t(P) \sim^{iid} B(P)$. We use different categories of VaR accuracy tests in this paper.

- Violation Ratio,
- Unconditional Coverage test,
- Independence test, and
- Dynamic Quantile test.

4.1. Violation Ratio

The VaR is violated if actual loss surpasses the VaR forecast. The violation ratio can be attained by diving the sum of actual exceedance by the expected number of exceedances given the forecasted period. The violation ratio is intended by (Danielsson, 2011)

$$Violation\ Ratio = \frac{E}{T * \alpha} \quad (4.2)$$

where,

E = Number of actual exceedances.

T= Total number of observations used to forecast VaR.

α= Probability level of VaR

Many risk managers agree that violation ratio between 0.8 and 1.2 is reliable, in the case when the violation ratio is less than 0.5 or exceeds 1.5, the model is unsatisfactory.

4.2. Unconditional Coverage Test

The first published statistical backtest was a coverage test (Kupiec, 1995). The test is an unconditional coverage (UC) test or Proportion of Failure (POF) test. Its null hypothesis indicates that the violated VaR percentage or failure rate P is consistent with the given confidence interval α. $H_0: P = \alpha$, v: is the sum of violations and f: is the length of the forecasting period.

$$UC = -2\left\{ln\left\{\frac{P^v(1-P)^{f-v}}{\hat{P}^v(1-\hat{P})^{f-v}}\right\}\right\} \quad (4.3)$$

The maximum likelihood estimator $\hat{P} = \frac{f}{v}$ under the alternative hypothesis. The null hypothesis was rejected as the critical value $\chi^2(1)$ exceeded on $\alpha\%$.

4.3. Independence of Violations:

The Kupiec test has concerned only with the coverage of VaR estimates, but it does not take account of the violation's clustering. Clustering of exceptions is something that helps VaR practitioners to detect large losses in the rapid successions by Christoffersen (1998). The independent test of Christofferson designed to detect the clustering of VaR violations. The likelihood ratio framework implemented under the null hypothesis of independence. The probability of today for the time t does not affect by the day before t, is $t-1$. $H_0: P_t = P_{t-1}$.

The independent test defined as:

$$IND = -2\left\{ln\left\{\frac{\hat{P}^v(1-\hat{P})^{f-v}}{\hat{P}_{12}^{v_{12}}(1-\hat{P}_{12})^{v_{12}}\hat{P}_{11}^{v_{11}}(1-\hat{P}_{11})^{v_{11}}}\right\}\right\} \quad (4.4)$$

where v_{ij} is a number of violations up to t and $t-1$. Independent test is not a complete test according to Christofferson, so he introduced the combination of unconditional coverage and the independent test, the conditional coverage CC-test presented as:

$$CC = UC + IND \quad (4.5)$$

Two separate statistics included in the CC-test, one unconditional coverage $UC = -2\{ln(L_1^{UC}) - ln(L_1^{UC})\}$ and other is $IND = 2\{ln(L_2^{IND}) - ln(L_2^{IND})\}$ (Braione and Scholtes, 2016). Campbell (2005) indicated that VaR model fails in unconditional coverage tests or in independent tests while it passes through the joint test. He suggested running the tests separately.

4.4. Dynamic Quantile (DQ) Test

Another test is a regression-based test introduced by Engle and Manganelli (2004) is dynamic Quantile (DQ) test. The test is associated with a Quantile process $H_t = I_t(\alpha) - \alpha$ resultant sequence is hit sequence, assumes the values,

$$H_t = \begin{cases} 1-\alpha & if\ I_t = 1 \\ -\alpha & if\ I_t = 0 \end{cases} \quad (4.7)$$

The indication of this approach regresses from present violations of the past violations, to examine the different restriction of the model parameters. The process of the DQ test is to test the joint hypothesis. $\beta_1, \beta_2, \beta_3, \cdots, \beta_k$. $H_0: \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_k = 0$, The assumption coincides with Christofferson's null hypothesis of the CC test. It is also possible to test the null hypothesis individually like individual coverage hypothesis and independent hypothesis.

$$H_0\ (DQ_{ind}): \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_k = 0$$
$$H_0\ (DQ_{cc}): \beta_1 = \beta_2 = \beta_3 = \cdots = \beta_k = 0$$

5. Results and Discussions

We use the data from yahoo finance: www.yahoofinance.com, daily stock indices of four Asian countries, Pakistan, India, Malaysia, and Singapore from 3/1/2008 to 31/12/2017 financial log-returns with daily prices. The daily log-returns of individual series are calculated on the logged difference of adjusted closed process for each country. The daily log returns are obtained by using the formula $r_{i,t} = 100[ln\ \ln(P_{i,t}) - \ln(P_{i,(t-1)})]$, where $P_{i,t}$ is the current day price and $P_{i,t-1}$ is the previous day's price.

The daily closing prices and log-returns of the Karachi, Bombay, Kuala Lumpur, and Singapore stock exchange are shown in Figure 1. High and low volatility and volatility clustering can be observed in the log-return series. Therefore, one of the key interests lies in modeling and forecasting volatility and risk of four Asian stock markets throughout this particular period.

The summary of descriptive statistics, the Jarque Bera test for normality, the Ljung Box test for autocorrelation and LM test for the log differenced returns is presented in Table 1. The analysis of the descriptive statistics reveals that except Singapore all the indexes had positive daily average returns. Jarque Bera normality test reveals that the log-returns of Pakistan, India, Malaysia, and Singapore are not normally distributed.

Moreover, the log-returns series of all four countries have an excess of kurtosis (P<0.001), which indicates that the distribution of these log-returns has heavier tails in contrast to the normal distribution. Furthermore the $QS(25)$ (Ljung-Box) test on squared log-returns at lag 25 is found highly significant indicating dependence in the squared log-returns that the log returns. The reported value of Engle's-LM test indicates that there is an ARCH effect up to lag 25, hence we can apply the multivariate GARCH model on the series of log returns.

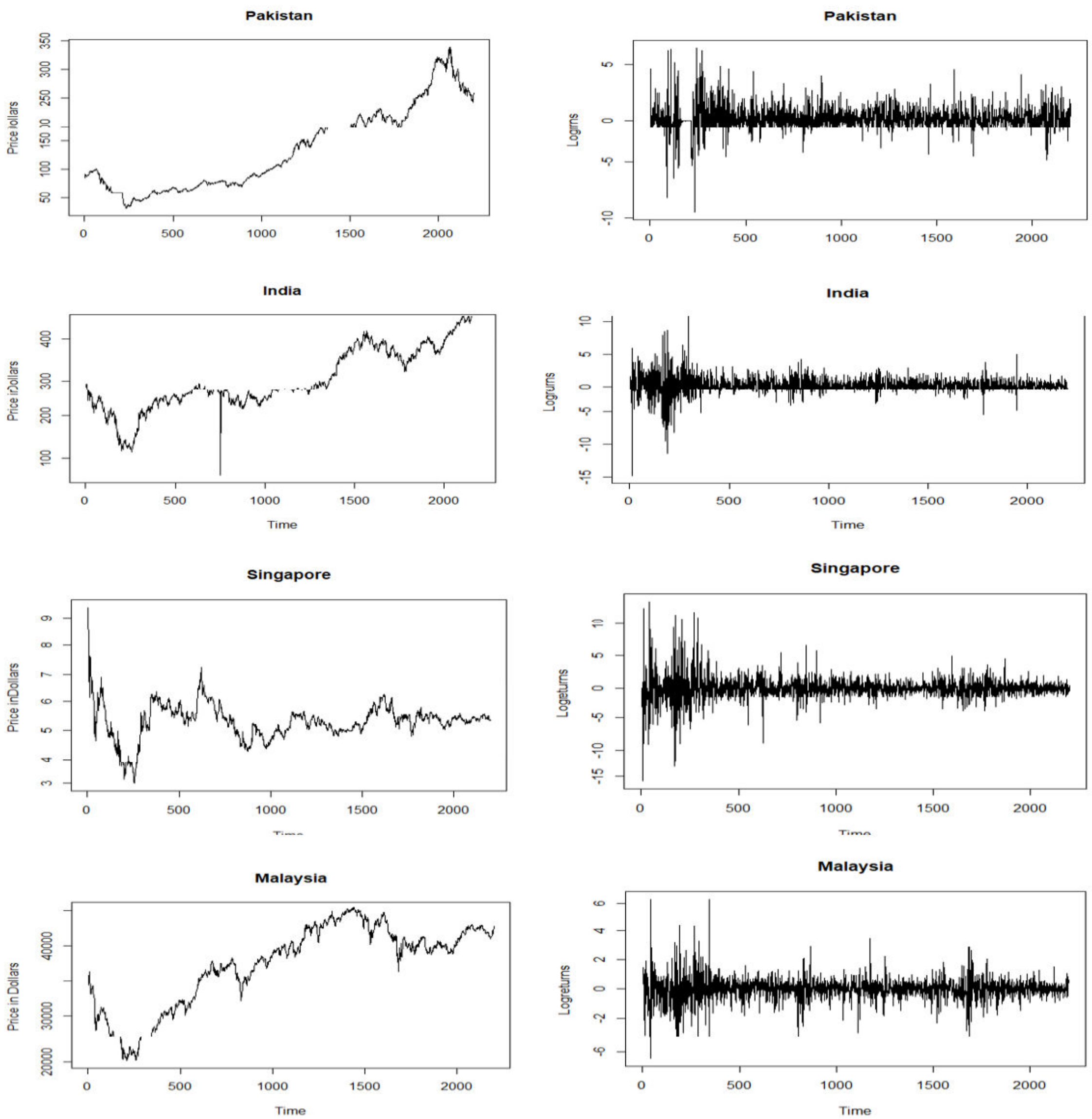

Figure 1. Daily Stock prices (Left) and returns (Right) of Karachi, Bombay, Singapore and Kuala Lumpur stock Exchanges.

Table 1. *Descriptive Summary of Log returns*

	Pakistan	*India*	*Malaysia*	*Singapore*
Mean	0.0511	0.0236	0.0100	-0.0262
Std.dev	1.2913	1.5883	0.7909	1.8422
Median	0.0406	0.0496	0.0165	0.0000
Minimum	-9.5191	-14.8116	-6.4482	-15.7185
Maximum	6.50652	12.0539	6.2831	13.6543
Skewness	-0.5192	-0.4884	-0.0655	0.2000
Kurtosis	5.4370	10.0809	9.0241	12.6978
J-B test	2815,1	9420.900	7481	14821
P-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
QS (25)	210.1280	352.6092	198.1394	283.6530
P-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)
LM	325.3900	27.7320	56.0937	439.5700
	(0.0000)	(0.0060)	(0.0002)	(0.0000)

Note: P-values in parenthesis. $QS(25)$ denotes the Ljung-Box test of squared residuals on lag 25 and LM denotes the Engle's Lagrange Multiplier test.

Table 2 represents the estimated parameters of the RDCC model, whereas all the parameters of the RDCC model for all three distributions are found to be statistically significant (P<0.001). The theoretical description of the RDCC model indicates that α *and* β greater than zero and $\alpha + \beta < 1$ ensures the positive definite and stationarity for the series of log returns. The reported values of parameters are all greater than zero whereas the sum of respective parameters α *and* β is also less than 1 for all four countries which indicates that the series of the log-returns in stationary, ergodic and have finite conditional variance.

Moreover, the direct comparison of model fit through Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) highlights that the models ensuring skewed distributions i.e. Student's-*t* distribution and multivariate Exponential Power distribution performed better than Normal distribution.

Table 2. *Full sample parameters estimates*

	RDCC-N	**RDCC-T**	**RDCC-MEP**
α_{pak}	0.1451***	0.1564***	0.1567***
	(0.0174)	(0.0123)	(0.0453)
β_{pak}	0.8141***	0.8155**	0.8126**
	(0.0239)	(0.0172)	(0.0669)
α_{ind}	0.0837*	0.0730**	0.0771*
	(0.0218)	(0.0390)	(0.2389)
β_{ind}	0.9086***	0.9216***	0.9167**
	(0.0250)	(0.0654)	(0.1209)
α_{mal}	0.1103**	0.1121***	0.1150**
	(0.0394)	(0.0123)	(0.0200)
β_{mal}	0.8647**	0.8698***	0.8643***
	(0.0529)	(0.0987)	(0.0830)
α_{sing}	0.0837***	0.0672***	0.0757**
	(0.0186)	(0.0432)	(0.0212)
β_{sing}	0.9115***	0.9291*	0.9203*
	(0.0206)	(0.0234)	(0.0111)
Alpha	0.0785*	0.0830**	0.0819**
	(0.0119)	(0.0877)	(0.0765)
beta	0.9934***	0.9931***	0.9935***
	(0.0024)	(0.0073)	(0.0043)
υ		6.9036**	
		(0.0367)	
Tail			1.9306***
			(0.0663)
Dynamic Parameters			
α	0.0061	0.0069	0.0067
β	0.9868	0.9862	0.9870
$\alpha + \beta$	0.9930	0.9931	0.9937
AIC	50557.564	49438.295	47371.220
BIC	50614.522	49552.210	47486.480
CAIC	50624.522	49572.210	47506.480
Log-likelihood	-25268.7822	-24699.1475	-25003.009
Residual Diagnostics			
QS (25)	11.8472	9.4863	7.8362
	(0.0864)	(0.6482)	(0.0832)

Note: Standard errors in Parenthesis. *, ** and *** denotes significance at 5%, 1% and 0.1% *** significance levels.

The Student's-*t* distribution outperforms with minimum AIC and BIC (AIC=35904.7940, BIC= 36020.054), whereas AIC and BIC were found very high for normal distribution (AIC= 50763.530, BIC= 50821.1 60). Hence the usage of skewed distribution assumption seems to be justified. The Ljung-Box test at lag 25 on squared residuals is not found significant at a 5% significance level for all models, which indicates the adequacy of the RDCC-GARCH model for four Asian Stock market data.

In-sample evaluation results

Data from 3rd January 2008 to 31st December 2015 (1500 observations), considered as an in-sample period, are utilized for in-sample VaR evaluation. The in-sample violation ratio and other backtesting results obtained from classical methods and RDCC models are reported in Table 3 and Table 4. All tests are computed at 1% and 5% VaR.

Table 3. *In-sample evaluation of classical methods*

1% VaR				
	Historical Simulation	Unconditional Parametric Method		
		Normal	Student's-*t*	MEP
Violation Ratio	0.9300	2.5023	2.096	2.7339
UC	3.5409	11.2489	12.9540	14.6352
	(0.430)	(0.002)	(0.003)	(0.005)
IND	12.0965	22.3053	17.7259	12.4390
	(0.0193)	(0.001)	(0.003)	(0.008)
CC	15.6374	33.5542	30.6779	27.0742
	(0.008)	(0.000)	(0.002)	(0.004)
DQ				
5% VaR				
Violation Ratio	1.9430	3.1104	2.0712	3.3000
UC	4.6071	15.3028	13.8701	9.2902
	(0.552)	(0.007)	(0.028)	(0.048)
IND	1.7810	8.4931	16.7207	12.0312
	(0.612)	(0.031))	(0.006)	(0.023)
CC	6.3881	23.8013	30.5908	21.3214
	(0.371)	(0.017)	(0.009)	(0.038)
DQ	18.3764	12.4392	26.8742	15.7772
	(0.013)	(0.033)	(0.009)	(0.023)

Note. P-values in parenthesis.

For 1% VaR, Historical simulation, RDCC-T, and RDCC-MEP have a more accurate violation ratio equal to 0.9300, 1.1890 and 1.3985, while violation ratio obtained from all other methods exceeds 2.

Table 4: *In-sample evaluation of RDCC-N, RDCC-T, and RDCC-MEP*

	1% VaR			5% VaR		
	RDCC-N	RDCC-T	RDCC-MEP	RDCC-N	RDCC-T	RDCC-MEP
Violation Ratio	2.9709	1.1890	1.3985	2.7837	1.1600	1.3808
UC	7.9182	1.3389	2.9831	15.6790	1.8791	2.9934
	(0.000)	(0.870)	(0.554)	(0.010)	(0.806)	(0.630)
IND	8.7651	2.7630	3.4872	14.9873	1.1845	1.7865
	(0.001)	(0.642)	(0.299)	(0.002)	(0.909)	(0.665)
CC	16.6833	4.1019	6.4703	27.6663	3.0636	4.7799
	(0.000)	(0.572)	(0.177)	(0.008)	(0.579)	(0.455)
DQ	15.5622	1.8547	7.9112	13.7689	2.9982	3.2140
	(0.008)	(0.716)	(0.007)	(0.029)	(0.881)	(0.237)

Note: P-values in parenthesis.

Many researchers and risk managers suggest that the violation ratio in the range of 0.8 to 1.2 assumes reliable accurateness of the model. In the case violation ratio greater than 1.5 or less 0.5, the model is imperfect. The UC test found non-significant for Historical simulation, RDCC-T, and RDCC-MEP whereas the results obtained from all other methods reject the null hypothesis which reveals that violated VaR is not consistent with the given confidence level. The IND test is accepted for Historical Simulation, RDCC-MEP, and RDCC-T which indicates that VaR violations are independently distributed. In the CC test, the RDCC model with heavy-tailed distribution showed better performance than other competing models. The DQ test is accepted for RDCC-T only, all other models contradict the null hypothesis of no higher-order dependence in VaR violation. Hence at 1% VaR RDCC-T outperforms all other approaches and less accurate methods amongst all these are unconditional parametric methods and RDCC-N for which none of the evaluation tests is accepted.

For 5% VaR forecasting, the violation ratio for RDCC-T and RDCC-MEP is 1.1600 and 1.3808 which was minimum among all other methods. The violation ratio for unconditional parametric methods was highest amongst all other methods. The UC, CC and IND tests are accepted for Historical simulation, RDCC-MEP, and RDCC-T, while for all other methods the results are found to be significant. The DQ test is accepted for RDCC-T and RDCC- MEP. For 5% VaR, we noted the best performance of the RDCC-T with the most reliable and lowest violation ratio and best results for evaluation tests followed by RDCC-MEP. Finally, for both 1% and 5% VaR the coverage tests evidenced the superiority of RDCC-T distribution with sufficiently accurate and minimum violation ratio. RDCC-MEP is also found a closer competitor to these models as compared to other methods RDCC-MEP provides consistent forecasting, the unconditional parametric methods and RDCC-GARCH model with a normal distribution of error performs worst at both confidence levels.

Out-of-sample evaluation results

The out-of-sample forecasting is valuable for investors, practitioners and for risk managers who desire to measure the model's performance based on risks forecast. In this section, we use estimation results to compute a one-step-ahead forecast at 1% and 5% confidence levels. Data from 1st January 2014 to 31st December 2017 (700 observations), considered as an out-of-sample period. The results of various evaluation tests for classical methods and RDCC models at 1% and 5% confidence levels are reported in Table 5 and Table 6.

Table 5: *Out-of-sample evaluation of classical methods*

	Historical Simulation	Unconditional Parametric Method		
		Normal	Student's-t	MEP
1%				
Violation Ratio	1.2083	2.5023	2.1076	2.7339
UC	11.2439	12.3852	11.4763	11.3848
	(0.003)	(0.003)	(0.002)	(0.002)
IND	13.4302	11.9430	22.8520	12.3131
	(0.009)	(0.001)	(0.000)	(0.001)
CC	24.6741	24.3282	34.3283	23.6979
	(0.008)	(0.006)	(0.000)	(0.009)
DQ	16.9430	21.4639	13.1038	13.9208
	(0.003)	(0.000)	(0.006)	(0.005)
5%				
Violation Ratio	1.702	2.110	2.0712	2.3000
UC	4.4708	12.7903	11.3980	13.8649
	(0.070)	(0.012)	(0.022)	(0.007)
IND	3.5896	12.4905	14.9826	8.2942
	(0.0743)	(0.036)	(0.037)	(0.028)
CC	8.0604	25.2808	26.3806	22.1591
	(0.027)	(0.007)	(0.006)	(0.039)
DQ	5.4764	16.4632	12.6303	11.0394
	(0.051)	(0.009)	(0.023)	(0.0429)

Note: P-values in parenthesis.

For 1% VaR the violation ratio for RDCC-T was found to be most accurate and lowest 1.0582 followed by Historical simulation, RDCC-MEP, and RDCC-N, the violation ratio for unconditional parametric methods exceed 2. To check the accuracy of these models UC test is applied, except for the RDCC-T model all other models contradict the null hypothesis which reveals what proportion of exception is less than 1% which indicates that these models overestimate the risk. The IND test is non-significant for RDCC-T and RDCC-MEP which indicates that the probability of exceptions occurring is equal, regardless of an exception that has occurred the day before. The CC test is found to be non-significant for RDCC-T and RDCC-MEP which reveals that the frequency of exceedance has consisted of forecasted VaR. The DQ test for higher-order dependence is rejected for all the models except RDCC-T. Hence at 1% VaR RDCC-T outdoes other competing models.

The out-of-sample results acquired from classical methods, RDCC-N, RDCC-T, and RDCC-MEP expose that violation ratio attained from RDCC-T at 5% was 1.0114, which was the lowest and most accurate among all the methods. The violation ratio for 5% found to be more accurate and less as compared to 1% VaR. In forecasting 5% VaR, RDCC-T computes best results for evaluation tests and we accept UC, IND, CC and DQ tests for this model. Again RDCC-N and Unconditional show poor performances, all evaluation tests for these methods are rejected at 5% VaR. Therefore, the overall VaR out-of-sample evaluation results reveal that the RDCC-GARCH model with Student's-*t* distribution provided the best VaR results at both confidence levels.

6. Conclusion

In this paper, we investigate the performance of RDCC-GARCH in forecasting one-day-ahead VaR using daily stock data of KSE100, BSE Ltd., KLSE and SGX from 2008 to 2017. We used Normal, Student's-*t* and Multivariate Exponential Power distribution of error for the estimation of VaR and the results were compared with Historical simulation and Unconditional VaR methods. We estimate one-day ahead VaR at 1% and 5% confidence levels. Of the set of specifications considered in this study, the classical methods and RDCC-GARCH with the normal distribution of error presented the worst performance, RDCC-MEP stood positively among the others for the in-sample period but the results were opposite for out- of sample forecast. Hence the overall study reveals that the RDCC-GARCH model with Student's-*t* errors is found superior in forecasting one-day-ahead VaR of four Asian stock markets. The findings of this study may help in providing accurate and reliable forecasts for high dimension asset returns.

Table 6: *Out-of-sample evaluation of RDCC-N, RDCC-T, and RDCC-MEP*

	1% VaR		
	RDCC-N	RDCC-T	RDCC-MEP
Violation Ratio	1.4509	1.0582	1.28142
UC	14.8834	1.9980	4.5432
	(0.000)	(0.092)	(0.009)
IND	8.9092	2.8881	3.9876
	(0.008)	(0.034)	(0.050)
CC	23.7926	4.8861	8.5308
	(0.000)	(0.014)	(0.0013)
DQ	24.0914	2.7892	13.7623
	(0.000)	(0.084)	(0.008)
	5% VaR		
	RDCC-N	RDCC-T	RDCC-MEP
Violation Ratio	2.9142	1.0114	1.8000
UC	2.9000	2.6291	6.9081
	(0.062)	(0.890)	(0.031)
IND	10.8790	3.0953	7.0478
	(0.007)	(0.054)	(0.014)
CC	20.779	5.7244	13.9559
	(0.000)	(0.343)	(0.003)
DQ	19.7357	4.3823	6.3480
	(0.000)	(0.459)	(0.239)

Note: P-values in parenthesis.

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