Math 4315 - PDEs   Sample Test 1

Test Date: Fri. Oct. 4, 2019

1. Solve the following first order PDEs by introducing an alternate coordinate system (i.e. $(x, y) \to (r, s)$)

$$
\begin{aligned}
(i) \quad & xu_x - yu_y &=& \quad 2u, \\
(ii) \quad & yu_x - u_y &=& \quad 1,
\end{aligned}
$$

2. Solve the following using the method of characteristics

$$
\begin{aligned}
(i) \quad & xu_x + (x + 2y)u_y &=& \quad u, \quad u(x,0) = x^2 \\
(ii) \quad & xu_x + 2uu_y &=& \quad u, \quad u(x,0) = x^2
\end{aligned}
$$

3. Solve the following nonlinear PDE

$$
\begin{aligned}
(i) \quad & xu_x^2 + u_y &=& \quad 1, \quad u(x,1) = x + 1. \\
(i) \quad u_x u_y - 2xu_x - 2yu_y &=& \quad 0, \quad u(x,0) = x^2.
\end{aligned}
$$