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Research Article

Certain Open Sets in Hereditary Generalized Topological Spaces

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Abstract

Characterization of sets in topological spaces has been carried out by many mathematicians over a period of time. However, characterizing hereditary topological spaces has not been exhausted. Certain aspects of these open sets include τ-open sets, τ-$\mathscr{H}$-open sets, and open functions among others. In this paper, we study in particular the concepts of τ-open sets and τ-$\mathscr{H}$-open sets. We also give some characterizations of τ-$\mathscr{H}$- continuous and τ-$\mathscr{H}$-open functions. Lastly, certain properties of τ-$\mathscr{H}$-open sets are outlined.

Keywords: τ-open sets; τ-$\mathscr{H}$-open sets; τ-$\mathscr{H}$-open functions; τ-continuous functions.

Introduction

Several concepts have been studied in topological spaces with very nice characterizations. Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years [1]. In [2] the authors defined continuous functions. In [3] the authors introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper, we define totally continuous functions and slightly continuous functions and basic properties of these functions are investigated and obtained. Throughout this work, (X, τ), (Y, σ) and (Z, *p*) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X). If A is open and closed, then it is said to be clopen (for details on clopen sets see [22-26]).

Since the introduction of M-topological spaces by [4], various authors [5], [6] and [7] have studied many other interesting topological properties in M- topological spaces. In [8] the authors studied b-open sets. In [9] the author studied the applications of b-connectedness. The authors in [10-16] studied generalized topology and modified generalized topology via hereditary classes. Several research papers have been published in recent years using γ-operator due to [17]. The notion of γ-open sets (originally called γ-sets) in topological spaces was introduced by [18].The generalization of open and closed set as like γ-open and γ-closed sets was introduced in [19] which is nearly to open and closed set respectively. These notion are plays significant role in general topology [20]. Moreover, the authors in [21-23] studied generalized open sets in Hereditary Generalized Topological Spaces (HGTS). In this paper, we study the concepts of τ-open sets and τ-$\mathscr{H}$-open sets. We also give some characterizations of τ -$\mathscr{H}$- continuous and τ -$\mathscr{H}$-open functions. Lastly, certain properties of τ -$\mathscr{H}$-open sets are outlined. So, the objective of the present paper is to extend the notion of general topological spaces to hereditary generalized topological spaces. We also study the concepts of τ-open sets and τ-$\mathscr{H}$-open sets in details. We also consider some characterizations of τ -$\mathscr{H}$- continuous and τ -$\mathscr{H}$-open functions.

Preliminaries

In this section we give some preliminary concepts which are useful in the sequel.

Definition 2.1.

Let X be a nonempty set and let expX be the power set of X. The collection τ of subsets of

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X satisfying the following conditions is called the generalized topology,
(i) $\phi \in \tau$,
(ii) $G_i \in \tau$ for $i \in I$ not equal ϕ implies $G = \bigcup_{i\in I} G_i \in \tau$.
The elements of τ are called the τ-open sets and their complements are called the τ-closed sets. The pair (X, τ) is called a generalized topological space (GTS).

Definition 2.2.

Let X be a nonempty set. A hereditary class $\mathscr{H}$ of X is defined as follows: If $A \in \mathscr{H}$ and $B \subseteq A$ then $B \in \mathscr{H}$. A generalized topological spaces (X, τ) with a hereditary class $\mathscr{H}$ is a hereditary generalized topological space and it is denoted by (X, τ, $\mathscr{H}$).

Definition 2.3.

Let (X, τ, $\mathscr{H}$) be a hereditary generalized topological space. For each $A \subseteq X$, $A^*(\mathscr{H}, \tau) = \{x \in X: A \cap G \in \mathscr{H}$ for every $G \in \tau$ such that $x \in G\}$. If there is no ambiguity then we write A^* in place of $A^*(\mathscr{H}, \tau)$. According to the definition, $x \in A^*$ if and only if there exists $x \in G \in \tau$ such that $(A \cap G)$ □ $\mathscr{H}$.

Definition 2.4.

Let (X, τ, $\mathscr{H}$) be a hereditary generalized topological space. For each $A \subseteq X$, $c_\tau^*(A) = A \cup A^*(\mathscr{H}, \tau)$.

Definition 2.5.

Let (X, τ, $\mathscr{H}$) be a hereditary generalized topological space. Any subset A of X is said to be p-q-open if $A \subseteq i(c_\tau(i_\tau(A)))$. The complement of a p-q-open set is said to be a p-q-closed set.

Definition 2.6.

Let (X, τ, $\mathscr{H}$) be a hereditary generalized topological space. Any subset A of X is said to be p-$\mathscr{H}$-open if $A \subseteq i(c_\tau^*(i_\tau(A)))$. The complement of a p-$\mathscr{H}$-open set is said to be a p-$\mathscr{H}$-closed set.

Results and discussion

Proposition 3.1.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) is said to be a τ -continuous function if $f^{-1}(A) \subseteq X$ is a τ_1-open set, for every τ_2-open set $A \subseteq Y$.

Proof:

Let X={a, b, c, d} and Y = {a, b, c}. Let τ_1= {ϕ, {a}, {b}, {a, b}} and τ_2= {ϕ, {a}}. Clearly, τ_1 and τ_2 are generalized topologies on X and Y respectively. Let $\mathscr{H}_1$={ϕ, {c}} and $\mathscr{H}_2$ = {ϕ, {b}} be the hereditary classes on X and Y respectively. Now, the triples (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) are hereditary generalized topological spaces. Let f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) be defined by f(a) = a, f(b) = a, f(c) = b and f(d) = c. For A = {a}$\in\tau_2$, $f^{-1}(A)$ ={a, b} is τ_1-open in (X, τ_1, $\mathscr{H}_1$). Similarly, for A = ϕ, $f^{1}(\phi)= \phi$ is also a τ_1-open in (X, τ_1, $\mathscr{H}_1$). Therefore, the inverse image of every τ_2-open set A is τ_1-open in (X, τ_1, $\mathscr{H}_1$). Hence, f is a τ-continuous function.

Theorem 3.2.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) is said to be an τ -$\mathscr{H}$-continuous function if $f^{-1}(A)$ is an τ -$\mathscr{H}$-open set in (X, τ_1, $\mathscr{H}_1$) for every τ_2-open set A of (Y, τ_2, $\mathscr{H}_2$).

Proof:

From Proposition 3.1., clearly the triples (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) are hereditary generalized topological spaces. Let f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) be any function defined as in Proposition 3.1. The collection of all τ -$\mathscr{H}$-open sets in (X, τ_1, $\mathscr{H}_1$) is {ϕ, {a}, {b}, {a, b}}. Clearly, the inverse image of every τ_2-open set in (Y, τ_2, $\mathscr{H}_2$) is τ-$\mathscr{H}$-open set in (X, τ_1, $\mathscr{H}_1$) Hence, f is a τ-$\mathscr{H}$-continuous function.

Theorem 3.3.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) is said to be an τ -$\mathscr{H}$-continuous function $f(A) \subseteq Y$ is a τ_2-open set, for every τ_2-open set $A \subseteq X$.

Proof:

From Proposition 3.1., clearly the triples (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) are hereditary generalized topological spaces. Let f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) be any function defined as in Proposition 3.1.
For A = {a}$\in\tau_1$, $f(A)$ ={a} is τ_2-open in (X, τ_2, $\mathscr{H}_2$). Similarly, for A = ϕ, f(A)= ϕ is also a τ_2-open in (X, τ_2, $\mathscr{H}_2$). Therefore, the inverse image of every τ_1-open set A is τ_1-open in (X, τ_1, $\mathscr{H}_1$)

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is a τ_2-open set in (X, τ_2, $\mathscr{H}_2$). Hence, f is a τ-continuous function.

Corollary 3.4.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) is said to be a τ-$\mathscr{H}$-open (resp., closed) function, if the image of each τ_1-open (resp., closed) set in (X, τ_1, $\mathscr{H}_1$) is a τ-$\mathscr{H}$-open (resp., closed) set in (Y, τ_2, $\mathscr{H}_2$).

Proof:

Let X={a, b, c, d} and Y = {a, b, c}. Let τ_1= {ϕ, {a}, {b}, {a, b}} and τ_2= {ϕ, {a}}. Clearly, τ_1 and τ_2 are generalized topologies on X and Y respectively. Let $\mathscr{H}_1$= {ϕ, {b}} and $\mathscr{H}_2$ = {ϕ, {c}} be the hereditary classes on X and Y respectively. Now, the triples (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) are hereditary generalized topological spaces. Let f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) be any function defined as in Proposition 3.1. The collection of all τ-$\mathscr{H}$-open sets in (Y, τ_2, $\mathscr{H}_2$) is {ϕ, {a}, {b}, {a, b}}. Here, the image of every -open set in (X, τ_1, $\mathscr{H}_1$) is a τ -$\mathscr{H}$-open set in (Y, τ_2, $\mathscr{H}_2$). Therefore, f is a τ-$\mathscr{H}$-open function. The following proposition whose proof is found in [4] is a useful result

Proposition 3.5.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Then for any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) the following statements are equivalent:

(i) f is an τ-$\mathscr{H}$-closed function;

(ii) f(A) is an τ -$\mathscr{H}$-closed set in (Y, τ_2, $\mathscr{H}_2$) for every τ-closed set A in (X, τ_1, $\mathscr{H}_1$);

(iii) $c(i_\tau(c_\tau^*(f(A))))\subseteq f(c_\tau(A))$.

Theorem 3.6.

Let (X, τ, $\mathscr{H}$) be a hereditary generalized topological space. Then (X, τ, $\mathscr{H}$) is said to be a τ-$\mathscr{H}$-$T_{1/2}$ space, if every τ-$\mathscr{H}$-open set is τ-open set in (X, τ, $\mathscr{H}$).

Proof:

From Proposition 3.1, clearly, the triplet (X, τ, $\mathscr{H}$) is a hereditary generalized topological space. The collection of all τ-$\mathscr{H}$- open sets in (X, τ, $\mathscr{H}$) is {ϕ, {a}, {b}, {a, b}}. Clearly, every τ-$\mathscr{H}$- open set is a τ-open set in (X, τ, $\mathscr{H}$). Hence, (X, τ, $\mathscr{H}$) is a τ-$\mathscr{H}$-$T_{1/2}$ space.

Corollary 3.7.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Then for any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) is a τ-$\mathscr{H}$-open function and if (Y, τ_2, $\mathscr{H}_2$) is τ-$\mathscr{H}$-$T_{1/2}$ space, then f is a τ-open function.

Corollary 3.8.

The statement of Corollary 3.7 is not valid if (Y, τ_2, $\mathscr{H}_2$) fails to be a τ-$\mathscr{H}$-$T_{1/2}$ space.

Proof:

Let X = {a, b, c, d, e, f } = Y. Let τ_1= { ϕ, { b, c, d, e } } and τ_2= { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d, e }, { a, c, d, e }, { a, b, c, d, e } } be the generalized topologies on X and Y respectively. Let $\mathscr{H}_1$= {ϕ, {a}} and $\mathscr{H}_2$ = {ϕ, {a}, {b}} be the hereditary classes on X and Y respectively. Clearly, (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) are hereditary generalized topological spaces. Let f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$) be an identity function. Here the collection of all τ-$\mathscr{H}$-open sets of (Y, τ_2, $\mathscr{H}$2) is { ϕ, { a }, { c }, { a, c }, { b, c }, { c, d }, { c, e }, { a, b, c },{ a, c, d }, { a, c, e }, { b, c, d }, { b, c, e }, { c, d, e }, { a, b, c, d }, { a, b, c, e }, { a, c, d, e }, { b, c, d, e }, { a, b, c, d, e } }. Clearly, (Y, τ_2, $\mathscr{H}_2$) is not a τ-$\mathscr{H}$-$T_{1/2}$ space and f is a τ-$\mathscr{H}$-open function. But f is not a τ-open function as there are some τ-$\mathscr{H}$-open sets in (Y, τ_2, $\mathscr{H}_2$) which are not τ_2-open in (Y, τ_2, $\mathscr{H}_2$). This clearly shows that Corollary 3.7 is valid only when (Y, τ_2, $\mathscr{H}_2$) is τ-$\mathscr{H}$-$T_{1/2}$ space.

Corollary 3.9.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Then for any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$), if f is a τ-$\mathscr{H}$-continuous function and (X, τ_1, $\mathscr{H}_1$) is a τ-$\mathscr{H}$-$T_{1/2}$ space, then f is a τ-continuous function.

Corollary 3.10.

Let (X, τ_1, $\mathscr{H}_1$) and (Y, τ_2, $\mathscr{H}_2$) be any two hereditary generalized topological spaces. Then for any function f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$), is a τ-$\mathscr{H}$-continuous function and g: (Y, τ_2, $\mathscr{H}_2$) → (Z, τ_3, $\mathscr{H}_3$), is a τ-continuous function, then g_0f: (X, τ_1, $\mathscr{H}_1$) → (Y, τ_2, $\mathscr{H}_2$), is an τ-$\mathscr{H}$-continuous function.

Conclusions

In the present paper, we have extended the notion of general topological spaces to

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hereditary generalized topological spaces. We studied the concepts of τ-open sets and τ-$\mathscr{H}$-open sets. We also gave some characterizations of τ -$\mathscr{H}$- continuous and τ -$\mathscr{H}$-open functions. Lastly, certain properties of τ -$\mathscr{H}$-open sets have also been outlined.

Conflicts of interest

Authors declare no conflict of interest.

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