Unsupervised image Segmentation Method With K-Means Clustering

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ABSTRACT:- This paper deals with the utilization of gamma distribution in image segmentation. The proposed algorithm is having application in security and surveillance analysis, etc. In some images the feature vector of image regions may not be measo kurtic and hence Gaussian mixture models may not suit well. Hence in this paper an image is developed and analyzed for image segmentation using finite mixture of gamma distribution and k-means clustering. The EM algorithm is utilized for obtaining the estimation of the model parameters. The model parameters are initialized with moment method of estimation and k-means clustering. The component maximum likely hood methods using Bayes principle segmentation is developed. The proposed algorithm performance is examined by computing image segmentation performance measures with an experimentation carried by choosing randomly five images from Berkeley data set. The performance measures revealed that this algorithm segment well the image regions than the existing segmentation methods for some images. The model parameters are estimated using EM-algorithm. The initialization of parameters is done with k-means algorithm and moment method of estimation.

Keywords:- Image segmentation , logistic distribution, EM-algorithm, performance evaluation.

I. INTRODUCTION

In the model based segmentation method the whole image is characterized by a mixture of probability distributions. The pixel intensity is considered as a feature for image segmentation. The pixel intensities in an image region may be distributed as platykurtic and laptokurtic, and mesokurtic. Due to simplicity and computational convenience, image segmentation algorithms based on Gaussian mixture models were developed(Yamazaki et al,(1998),T.Lie et al(1993), N.Nasios et al(2006),Z.H.Zhang et al (2003)). However in Gaussian mixture models the pixel intensities in image region are meso kurtic. But in many image regions the pixel intensities may not be distributed as mesokurtic,. Recently Srinivara Rao et al(2018) have introduced a logistic distribution which is very useful in portraying symmetric and platykurtic distributions. In this paper we develop and analyze the image segmentation algorithm using gamma distribution.

II. GAMMA DISTRIBUTION

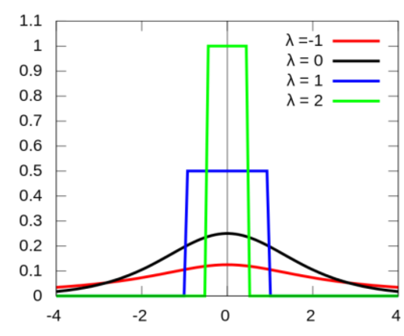
In this section, we briefly present the gamma distribution. In each image region the image data is quantified by pixel intensities**.** This section deals with the methodology for obtaining estimates of the parameters involved in the model through Expectation and Maximization algorithm (Mclanchlan G and Krishnan T (1997)). The image region pixel intensities are considered as features of the image. Here the gamma distribution is assumed for modeling the image region pixel intensities. As a result of it the whole image can be characterized with a gamma mixture model. The probability density function (p.d.f) of the pixel intensities as of the form

The probability density function of the pixel intensity is given by

$$f\left(z,k,p,l,m\right)=\frac{p\left(z-l\right)^{pk-1}}{e^{-\left(\frac{(z-l)}{m}\right)^{p}}m^{pk}\Gamma\left(k\right)}$$

l- is the location parameter, m-shape parameter, p,l,m,k are the gamma variants.

The frequency curve associated with gamma distribution is shown in figure



The entire image is a collection of regions which are characterized by gamma distribution . Hence, it is assumed that the pixel intensities of the whole image follows k-component mixture of gamma distribution and its probability density function is of the form.

$$p(x)=\sum_{i=1}^{k}\alpha_i f_i(x,\mu,\sigma^2) \quad (2)$$

Where k is the number of regions $0 \le \alpha_i \le 1$ are weights such that $\sum \alpha_i = 1$ and $f_i(x,\mu,\sigma^2)$ is given in equation (1). α_i is the weight associated with i^th region in the whole image.

In general the pixel intensities in the regions are statically correlated and these correlations can be reduced by spatial sampling (Lei T. and Sewehand W. (1992)) or spatial averaging (Kelly P.A. et al (1998)). After reduction of correlation, the pixels are considered to be uncorrelated and independent. The mean pixel intensity of the whole image is

$$E(X)=\sum_{i=1}^{K}\alpha_i \mu_i$$

III. ESTIMATION OF MODEL PARAMETERS USING EM ALGORITHM

The parameters of the model are estimated by using likelihood function of the sample observations. The likelihood equations are usually found by differentiating the logarithm of likelihood function, setting the derivatives equal to zero, and perhaps performing some additional algebraic manipulations. For this distribution, the likelihood equation is nonlinear and there is no solution by analytic means. Consequently, we use some iterative procedure like EM algorithm for obtaining the estimates of the parameters.

The updated equations of the model parameters are obtained for Expectation Maximization (EM) algorithm.

The likelihood of the function of model, is

$$L(\theta)=\prod_{S=1}^{N} p(x_s,\theta^{(l)}) \quad (3)$$

$$L(\theta)=\prod_{s=1}^{N}\left(\sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)}\right) \quad (4)$$

This implies

$$\log L(\theta)=\sum_{S=1}^{N}\log\left(\sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)})\right)$$

Where $\theta=(\mu_i,\sigma_i^2,\alpha_i; i=1,2,................k)$ is the set of parameter

Therefore

$$\log L(\theta)=\sum_{S=1}^{N}\log\left[\sum_{i=1}^{k}\alpha_i \frac{p(z-l)^{pk-1}}{e^{-\left(\frac{(z-l)}{m}\right)^p} m^{pk}\Gamma(k)}\right] \quad (5)$$

The first step of the EM algorithm requires the estimation of the likelihood function of sample observations

E-STEP:-

In the expectation (E) step, the expectation value of log $L(\theta)$ with respect to the initial parameter vector $\theta^{(0)}$ is

$$Q(\theta,\theta^{(0)})=E_{\theta^{(0)}}\left[\log L(\theta)/\bar{x}\right] \quad (6)$$

Given the initial parameters $\theta^{(0)}$. One can compute the density of pixel intensity X as

$$P(x_s,\theta^{(l)})=\sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)}) \quad (7)$$

$$L(\theta)=\prod_{S=1}^{N} p(x_s,\theta^{(l)}) \quad (8)$$

This implies

$$\log L(\theta)=\sum_{S=1}^{N}\log\left(\sum_{i=1}^{k}\alpha^{(l)}{}_i f_i(x_s,\theta^{(l)})\right)$$

The conditional probability of any observations x_s, belongs to any region 'k' is

$$P_k(x_s,\theta^{(l)})=\left[\frac{\alpha_k^{(l)} f_k(x_{s,}\theta^{(l)})}{p_i(x_s,\theta^{(l)})}\right] \quad (10)$$

$$p_k(x_s,\theta^{(l)}) = \left[\frac{\alpha_k^{(l)} f_k(x_{s,}\theta^{(l)})}{\sum_{i=1}^{k} \alpha_i^{(l)} f_i(x_s,\theta^{(l)})} \right] \quad (11)$$

$$p_k(x_s,\theta^{(l)}) = \left[\frac{\alpha_k^{(l)} f_k(x_{s,}\theta^{(l)})}{\sum_{i=1}^{k} \alpha_i^{(l)} f_i(x_s,\theta^{(l)})} \right] \quad (11)$$

The expectation of the log likelihood function of the sample is

$$Q(\theta,\theta^{(l)}) = E_{\theta^{(l)}} \left[\log L(\theta) / \bar{x} \right]$$

But we have

$$f_i(x_s,\theta^{(l)}) = \frac{e^{\left[\frac{-(x_s-\mu_i^{(l)})}{\sigma^{(l)}}\right]}}{\sigma^{2(l)} \left(1+e^{\left[\frac{x_s-\mu_i^{(l)}}{\sigma^{(l)}}\right]}\right)^2} \quad (12)$$

Following the heuristic arguments of Jeff A.Bilmes(1997) we have

$$Q(\theta,\theta^{(l)}) = \sum_{i=1}^{k}\sum_{s=1}^{N} \left(P_i(x_s,\theta^{(l)})(\log f_i(x_s,\theta^{(l)}) + \log \alpha_i^{(l)}) \right) \quad (13)$$

M-STEP:-

For obtaining the estimation of model parameters one has to maximize $Q(\theta,\theta^{(l)})$ such that $\sum \alpha_i = 1$. This can be solved by applying the standard solution method for constrained maximum by constructing the first order Lagrange type function

$$F = \left[E\left(\log L(\theta^{(l)})\right) + \lambda \left(1 - \sum_{i=1}^{k} \alpha_i^{(l)} \right) \right] \quad (14)$$

Where, λ is Lagrangian multiplier combining the constraint with the log likelihood functions to be maximized.

The above two steps are repeated as necessary, each iteration is guaranteed to increase the loglikehood and the algorithm is guaranteed to converge to a local maximum of the likelihood function

The mean of gamma distribution:

$$m^2 \frac{\Gamma\left(p+\frac{2}{k}\right)}{\left[\Gamma(p)\right]^2} = \frac{\Gamma\left(p+\frac{1}{k}\right)}{\left[\Gamma(p)\right]^2}$$

And i[th] moment of location parameter $m^k \frac{\Gamma\left(p+\frac{i}{k}\right)}{\left[\Gamma(p)\right]^2}$

IV. INITIALIZATION OF THE PARAMETERS BY K-MEANS

The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of regions in the image. The number of mixture components taken for k-means algorithm is obtained, by plotting the histogram of the pixel intensities of the whole image, and the number of peaks in the histogram can be taken as the initial value of the number of regions k.

The parameters α_i , μ and σ^2 are usually considered as known apriori. A commonly used method in initializing parameters is by drawing a random sample from the entire image (McLachan G. AND Peel D.(2000). This method performs well, if the sample size is small, some small regions may not be sampled.

To overcome this problem we use the k-means algorithm to divide the whole image into homogeneous regions. In k-means algorithm the centroids of the clusters are recomputed as soon as the pixel joins a cluster.

The k-means algorithm is one of the clustering techniques for which the objective is to find the partition of the data which minimizes the squared distances between all points and their respective cluster centers (Rose H.Turi,(2001)).

k-means algorithm uses an iterative procedure that minimizes the sum distances from each object to its cluster centroid, over all clusters. This procedure consists of the following steps.

1. Randomly choose K data points from the dataset as initial clusters. These data points represent initial cluster centroids.
2. Calculate Euclidian distance of each data point from each cluster centre and assign the data points to its nearest cluster center.

3. Calculate new cluster center so that squared error distance of each cluster should be minimum.
4. Repeat step 2 and 3 until clustering centers do not change.
5. Stop the process.

In the above algorithm the cluster centers are only updated once all points have been allocated to their closed cluster center. k-means algorithm depends on the parameter k, the number of clusters in the image.

After determining the final values of k(number of regions),we obtain the initial estimates of μ_i, σ_i^2 and α_i for the i[th] region using the segmented region pixel intensities with the method given by Srinivasa Rao K, et.al., (1997) for two parameter logistic distribution .

The initial estimate as $\alpha_i = \frac{1}{k}$, where i=1,2,3………………k. The parameter μ_i and σ_i^2 are estimated by the method of moments as $\hat{\mu}_i = \bar{X}$ and $\sigma_i^2 = \frac{4n_i}{3(n_i - 1)} S^2$, where S^2 is sample variance, n_i is the number of observations in the i[th] segmentation.

4.2 SEGMENTATION ALGORITHEM

In this section, we present the image segmentation algorithm. After refining the parameters, the prime step in image segmentation on allocating the pixels to the segments of the image. This operation is performed by segmentation algorithm. The image segmentation algorithm consists of four steps.

Step 1) Plot the histogram of the whole image.

Step 2) Obtain the initial estimates of the model parameters using K-means algorithm And moment estimates for each image region as discussed in section 1.4.

Step 3) Obtain the refined estimates of the model parameters μ_i, σ_i^2 and α_i for i=1,2,3……k, Using the EM algorithm with the updated equations given by (5),(7),And (8) respectively in section 3.3.

Step 4) Assign each pixel into the corresponding j[th] region (segment) according to the maximum likelihood of the j[th] component L_j

That is

$$L_j = MAX\left[\frac{p(z-l)^{pk-1}}{e^{-\left(\frac{(z-l)}{m}\right)^p} m^{pk}\Gamma(k)}\right], -\infty < z_s < \infty,$$

$$-\infty < \mu_j < \infty, \sigma_j > 0$$

V. EXPERIMENTATION AND RESULTS

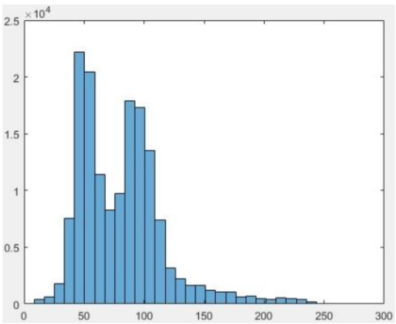
The EM algorithm for model has been implemented in MATLAB and tested its efficiency for image segmentation. To demonstrate the utility of the image segmentation algorithm developed, an experiment is conducted with five images taken from Berkeley

Images data. The images WOMAN,TREE, DEAR,STAR FISH, and OSTRICH are considered for image segmentation. The pixel intensities of the image are assumed to follow a logistic distribution. We consider that the image contains k regions and pixel intensities in each image region follow a gamma distribution with different parameters. The number of segments in each of five images considered for experimentation is determined by the histogram of pixel intensities. The histograms of the pixel intensities of the five images are shown in figure 1.2

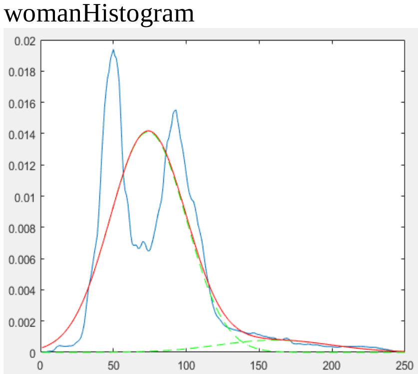
WOMAN ,Tree,Dear,Star Fish, And Ostrich.

Quality metric	**Formula to Evaluate**
Average Difference (AD)	$\sum_{j=1}^{M}\sum_{k=1}^{N}[F(j,k) - \hat{F}(j,k)] / MN$ Where M,N are image matrix rows and columns
Maximum Distance (MD)	Max{|
Image Fidelity(IF)	$1 - \left[\sum_{j=1}^{M}\sum_{k=1}^{N}[F(j,k) - \hat{F}(j,k)]^2 / \sum_{j=1}^{M}\sum_{k=1}^{N}[F(j,k))]^2\right]$ Where M,N are image matrix rows and columns
Mean Squared error (MSE)	$\frac{1}{MN}\sum_{j=1}^{M}\sum_{k=1}^{N}[O\{F(j,k)\} - O\{\hat{F}(j,k)\}]^2 / \sum_{j=1}^{M}\sum_{k=1}^{N}[O\{F(j,k)^2$ Where M,N are image matrix rows and columns
Signal to noiseratio (SNR)	$20.\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$ Where, MAX_I is maximum possible pixel value of image, MSE is the Mean squared error

Woman grey image, woman segmented image

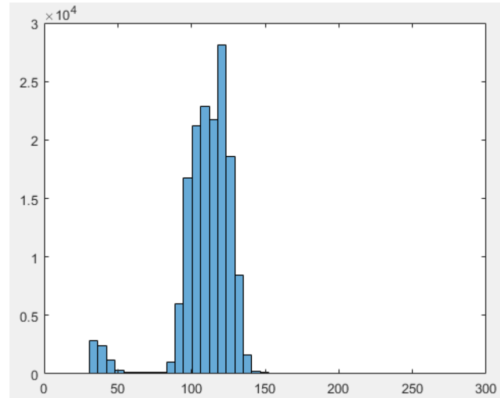


womanHistogram

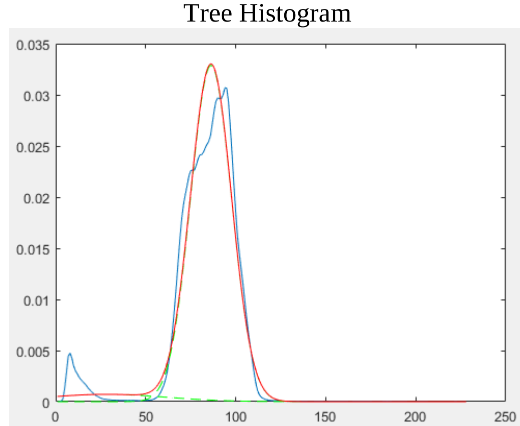


Plots of woman Probability Density and logistic Estimated by EM

TREE GREY IMAGE TREE SEGMENTEDIMAGE



Tree Histogram

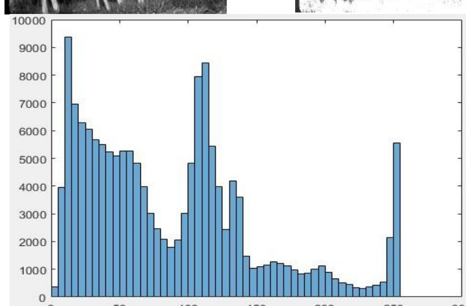


Plots of Tree **Probability Density logistic Estimated by EM**

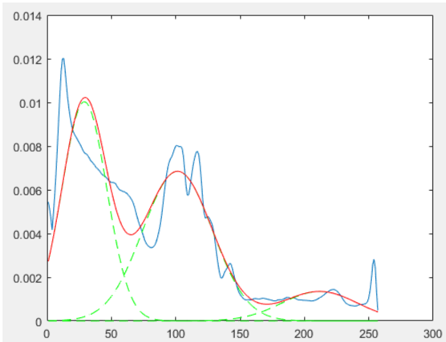
Table: logistic ML Estimates for WOMAN data for (K=3)

Deer Gray Image

Deer Segment



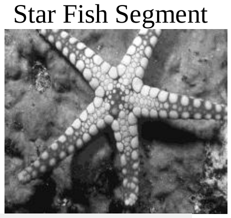
Deer **Histogram**

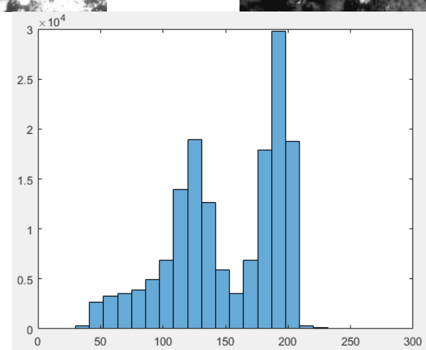


Plots of Deer **Probability Density logistic Estimated by EM**

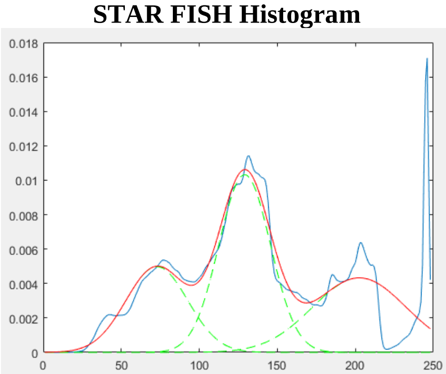
Table5.1d: ML Estimates for STAR FISH data for (K=3)

Star Fish Gray Image Star Fish Segment





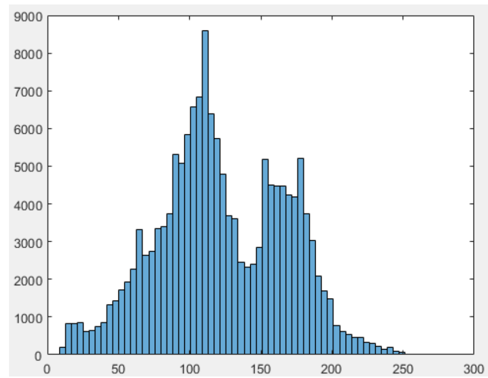
STAR FISH Histogram



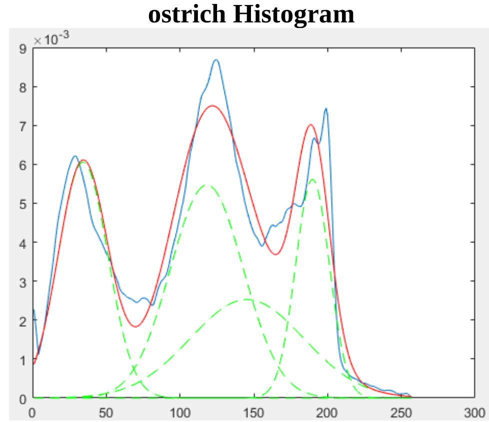
Plots of STAR FISH Probability Density logistic Estimated by EM

Table5.1e: ML Estimates for Tiger data for (K=3)

Ostrich grey image, ostrich segmented image



ostrich Histogram



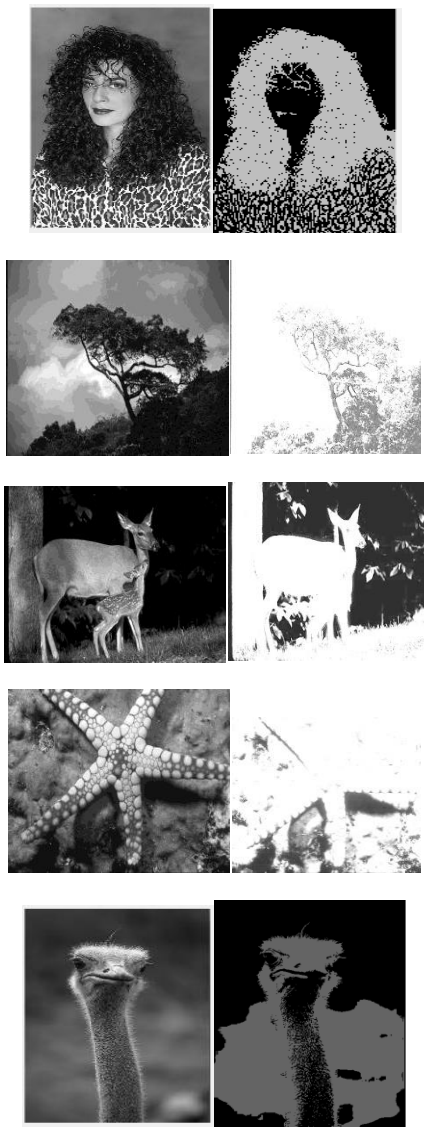
Plots of ostrich Probability Density of logistic Estimated by EM

Substituting the final estimates of the model parameters, the probability density function of pixel intensities of each image is estimated

Using the estimated probability density function and image segmentation algorithm given in section 2.1, the image segmentation is done for the five images under consideration, The original and segmented images are shown in Figure 5.2

5.2 Original and Segmented Image

ORIGINAL IMAGES SEGMENTEDIMAGES



VI. PERFORMANCE EVALUATION

After conducting the experiment with the image segmentation algorithm developed in this paper, its performance is studied. The performance evaluation of the segmentation technique is carried by obtaining the three performance measures namely,(i) probabilistic rand index(PRI), (ii) variation of information(VOI) and (iii) global consistence error(GCE).

The computed values of the performance measures for the new and the earlier existing finite Gaussian mixture model (GMM) with k-means algorithm are presented in the Table for a comparative study

From the Table 6.3, it is observed that all the image quality measures for the five images are meeting the standard criteria. This implies that using the proposed algorithm the images are retrieved accurately.

VII. COMPARATIVE STUDY

A comparative study of the proposed algorithm with that of the algorithms based on finite Gaussian mixture model reveals that the MSE of the proposed model is less than that of finite Gaussian mixture model. Based on all other quality metrics.it isalso observed that the performance of the proposed model in retrieving the images is better than the Gaussian mixture model.

IMAGES	**Quality Metrics**	**GMM**	**gamma**	**Standard Limits**	
	Average Difference	0.456	0.76	-1 to 1	Closer to 1
	Maximum Distance	0.345	0.879	-1 to 1	Closer to 1
	Image Fidelity	0.44	0.86	0 to 1	Closer to 1
	Mean Squared error	0.22	0.23	0 to 1	Closer to 0
	Signal to Noise ratio	19.88	37.98	-∞ to ∞	As big Possible
	Average Difference	0.231	0.473	-1 to 1	Closer to 1
	Maximum Distance	0.224	0.977	-1 to 1	Closer to 1

	Image Fidelity	0.212	0.813	0 to 1	Closer to 1
	Mean Squared error	0.24	0.121	0 to 1	Closer to 0
	Signal to Noise ratio	21.42	33.28	$-\infty$ to ∞	As big Possible
	Mean Squared error	0.2514	0.228	0 to 1	Closer to 0
	Signal to Noise ratio	3.241	5.514	$-\infty$ to ∞	As big Possible
	Average Difference	0.21	0.3653	-1 to 1	Closer to 1
	Maximum Distance	0.21	0.892	-1 to 1	Closer to 1
	Image Fidelity	0.2134	0.787	0 to 1	Closer to 1
	Mean Squared error	0.06	0.145	0 to 1	Closer to 0
	Signal to Noise ratio	13.43	49.22	$-\infty$ to ∞	As big Possible
	Average Difference	0.3232	0.322	-1 to 1	Closer to 1
	Maximum Distance	0.123	0.212	-1 to 1	Closer to 1
	Image Fidelity	0.233	0.897	0 to 1	Closer to 1
	Mean Squared error	0.01	0.4345	0 to 1	Closer to 0
	Signal to Noise ratio	11.11	27.267	$-\infty$ to ∞	As big Possible

VIII. CONCLUSION

This paper deals with a novel application of gamma distribution in image segmentation. The whole image is characterized by a finite mixture of gamma distribution. The gamma distribution includes a family of platykurtic distributions. The updated equations of the model parameters are derived and solved using mat-lab code. The initialization of parameters is done through k-means algorithm and moment method of estimation .An experimentation with five images taken from Berkeley-image database revealed that the proposed algorithm performs better with respect to image segmentation metric that image segmentation metrics method of GMM. The hybridization of model based approach with k-means improves the efficiency of segmentation .The proposed algorithm can be further extended to the other method of estimation of the model parameters such as Monto-Carlo methods and Bootstrapping methods which will be taken elsewhere

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