Chapter 3
Quadratic Equations and Complex Numbers

Section 3-1
Solving Quadratic Equations

Simplifying Square Roots

Example 1 Simplify $\sqrt{8}$.

$\sqrt{8} = \sqrt{4 \cdot 2}$ — Factor using the greatest perfect square factor.

$= \sqrt{4} \cdot \sqrt{2}$ — Product Property of Square Roots

$= 2\sqrt{2}$ — Simplify.

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

Example 2 Simplify $\sqrt{\frac{7}{36}}$.

$\sqrt{\frac{7}{36}} = \frac{\sqrt{7}}{\sqrt{36}}$ — Quotient Property of Square Roots

$= \frac{\sqrt{7}}{6}$ — Simplify.

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

Simplify the expression.

1. $\sqrt{27}$ **2.** $-\sqrt{112}$ **3.** $\sqrt{\frac{11}{64}}$ **4.** $\sqrt{\frac{147}{100}}$

Factoring Special Products

Example 3 **Factor (a) $x^2 - 4$ and (b) $x^2 - 14x + 49$.**

a. $x^2 - 4 = x^2 - 2^2$ — Write as $a^2 - b^2$.

$= (x + 2)(x - 2)$ — Difference of Two Squares Pattern

▶ So, $x^2 - 4 = (x + 2)(x - 2)$.

b. $x^2 - 14x + 49 = x^2 - 2(x)(7) + 7^2$ — Write as $a^2 - 2ab + b^2$.

$= (x - 7)^2$ — Perfect Square Trinomial Pattern

▶ So, $x^2 - 14x + 49 = (x - 7)^2$.

Factor the polynomial.

10. $x^2 - 9$

11. $4x^2 - 25$

13. $x^2 + 28x + 196$

14. $49x^2 + 210x + 225$

Solving Quadratic Equations by Graphing

A **quadratic equation in one variable** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. A **root of an equation** is a solution of the equation. You can use various methods to solve quadratic equations.

STUDY TIP
Quadratic equations can have zero, one, or two real solutions.

Core Concept

Solving Quadratic Equations

By graphing	Find the x-intercepts of the related function $y = ax^2 + bx + c$.
Using square roots	Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.
By factoring	Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

Tutorial

EXAMPLE 1 Solving Quadratic Equations by Graphing

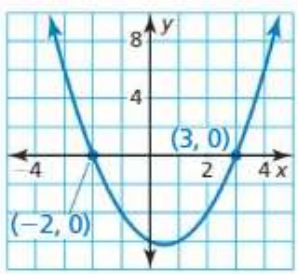
Solve each equation by graphing.

a. $x^2 - x - 6 = 0$

b. $-2x^2 - 2 = 4x$

SOLUTION

a. The equation is in standard form. Graph the related function $y = x^2 - x - 6$.



The x-intercepts are -2 and 3.

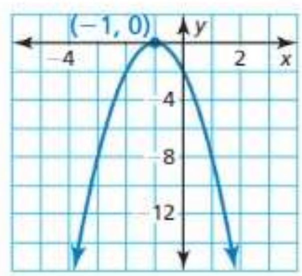
▶ The solutions, or roots, are $x = -2$ and $x = 3$.

Check

$$x^2 - x - 6 = 0$$
$$(-2)^2 - (-2) - 6 \stackrel{?}{=} 0$$
$$4 + 2 - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$$x^2 - x - 6 = 0$$
$$3^2 - 3 - 6 \stackrel{?}{=} 0$$
$$9 - 3 - 6 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

b. Add $-4x$ to each side to obtain $-2x^2 - 4x - 2 = 0$. Graph the related function $y = -2x^2 - 4x - 2$.



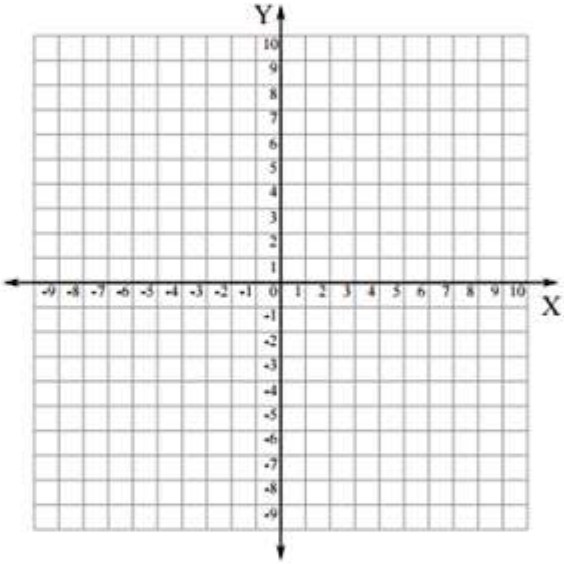
The x-intercept is -1.

▶ The solutions, or roots is $x = -1$.

I Would Not Recommend This Method. Solve Quadratic Equations Algebraically Instead

Solve the equation by graphing.

1. $x^2 - 8x + 12 = 0$



X	Y

Solving Quadratic Equations Algebraically

When solving quadratic equations using square roots, you can use properties of square roots to write your solutions in different forms.

When a radicand in the denominator of a fraction is not a perfect square, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called *rationalizing the denominator*.

EXAMPLE 2 Solving Quadratic Equations Using Square Roots

Solve each equation using square roots.

a. $4x^2 - 31 = 49$ **b.** $3x^2 + 9 = 0$ **c.** $\frac{2}{5}(x + 3)^2 = 5$

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Remember that soving an equation, finding the x-intercept, finding the zeros, and finding the solutions all mean the same thing when refering to quadratic functions.

Solve the equation by factoring.

7. $x^2 + 12x + 35 = 0$

8. $3x^2 - 5x = 2$

Find the zero(s) of the function.

9. $f(x) = x^2 - 8x$

10. $f(x) = 4x^2 + 28x + 49$

Solving Real-Life Problems

To find the maximum value or minimum value of a quadratic function, you can first use factoring to write the function in intercept form $f(x) = a(x - p)(x - q)$. Because the vertex of the function lies on the axis of symmetry, $x = \frac{p + q}{2}$, the maximum value or minimum value occurs at the average of the zeros p and q.

EXAMPLE 5 Solving a Multi-Step Problem

A monthly teen magazine has 48,000 subscribers when it charges $20 per annual subscription. For each $1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

SOLUTION

Step 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

Step 2 Write a verbal model. Then write and simplify a quadratic function.

Annual revenue (dollars)	=	Number of subscribers (people)	•	Subscription price (dollars/person)
$R(x)$	=	$(48{,}000 - 2000x)$	•	$(20 + x)$

$$R(x) = (-2000x + 48{,}000)(x + 20)$$

$$R(x) = -2000(x - 24)(x + 20)$$

Step 3 Identify the zeros and find their average. Then find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 24 and -20. The average of the zeros is $\frac{24 + (-20)}{2} = 2$.

To maximize revenue, each subscription should cost $20 + $2 = $22.

Step 4 Find the maximum annual revenue.

$$R(2) = -2000(2 - 24)(2 + 20) = \$968{,}000$$

▶ So, the magazine should charge $22 per subscription to maximize annual revenue. The maximum annual revenue is $968,000.

Section 3-1 Homework #13,15,17, 23, 27, 29, 31, 33, 35, 39, 41, 43, 47, 49, 51, 53, 69, 79