Classifying Direct and Inverse Variation

$y = ax$

You have learned that two variables x and y show direct variation when $y = ax$ for some nonzero constant a. Another type of variation is called *inverse variation*.

Core Concept

Inverse Variation

Two variables x and y show **inverse variation** when they are related as follows:

$$y = \frac{a}{x},\ a \neq 0$$

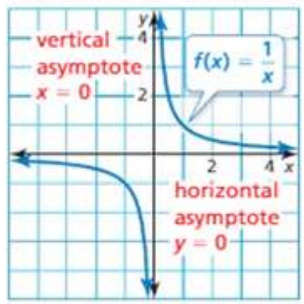
The constant a is the **constant of variation**, and y is said to *vary inversely* with x.

Core Concept

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.



Core Concept

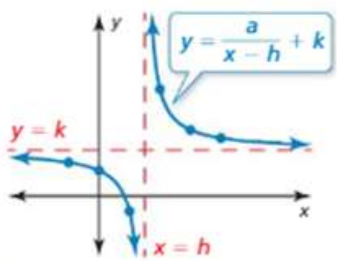
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x-h} + k$, follow these steps:

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.



Graphing Other Rational Functions

All rational functions of the form $y = \frac{ax+b}{cx+d}$ also have graphs that are hyperbolas.

- The vertical asymptote of the graph is the line $x = -\frac{d}{c}$ because the function is undefined when the denominator $cx + d$ is zero.
- The horizontal asymptote is the line $y = \frac{a}{c}$.

Core Concept

Simplifying Rational Expressions

Let a, b, and c be expressions with $b \neq 0$ and $c \neq 0$.

Property $\frac{ac}{bc} = \frac{a}{b}$ — Divide out common factor c.

Examples $\frac{15}{65} = \frac{3 \cdot 5}{13 \cdot 5} = \frac{3}{13}$ — Divide out common factor 5.

$\frac{4(x+3)}{(x+3)(x+3)} = \frac{4}{x+3}$ — Divide out common factor $x + 3$.

Core Concept

Multiplying Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ — Simplify $\frac{ac}{bd}$ if possible.

Example $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{10 \cdot 3 \cdot x \cdot x^2 \cdot y^3}{10 \cdot 2 \cdot x \cdot y^3} = \frac{3x^2}{2}, \quad x \neq 0, y \neq 0$

Core Concept

Dividing Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Property $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ — Simplify $\frac{ad}{bc}$ if possible.

Example $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}, x \neq \frac{3}{2}$

Core Concept

Adding or Subtracting with Like Denominators

Let a, b, and c be expressions with $c \neq 0$.

Addition

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Subtraction

$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

Core Concept

Adding or Subtracting with Unlike Denominators

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad+bc}{cd}$

Subtraction

$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad-bc}{cd}$

Core Concept

Simplifying Complex Fractions

Method 1 If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

Method 2 Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.