Comparative Analysis of Haar, Daubechies and Bior wavelets on Image Compression using Discrete Wavelet Transform

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***Abstract* -** Image compression plays a significant role in many fields, such as medical diagnosis, satellite remote sensing, telecommunication industry, high definition television and robotics. The aim of image compression is to reduce the size of an image but without losing the quality and information from image. Digital images are very large in size and hence occupy larger storage space. Due to their larger size, they take larger bandwidth and more time for upload or download through the Internet. This makes it inconvenient for storage as well as file sharing. To combat with this problem, the images are compressed in size with various techniques. Wavelet transform is designed in such a way that it provides good frequency for low frequency components and high temporal resolution for high frequency components. This paper represents the analysis for various image compression techniques using wavelet transform. The results through various compression techniques are compared.

***Keywords* -** *Image compression, Discrete Wavelet Transform (DWT), Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR, Energy Retained (ER), and Compression Ratio (CR).*

I. INTRODUCTION

Image Compression, is an art and science of reducing the amount of data required to represent an image. One of the most useful and commercially successful technologies in the field of image processing. The number of images that are compressed and decompressed daily is staggering, and the compressions and decompressions themselves are virtually invisible. Image compression is important for many applications that involve huge data storage, transmission and retrieval such as for multimedia, documents, videoconferencing, and medical imaging. Uncompressed images require considerable storage capacity and transmission bandwidth. The objective of image compression technique is to reduce redundancy of the image data in order to be able to store or transmit data in an efficient form. This results in the reduction of file size and allows more images to be stored in a given amount of disk or memory space.

Wavelet Transform is a type of signal representation that can give the frequency content of the signal at a particular instant of time. Wavelet transform is designed in a way that it provides good frequency for low frequency components and high temporal resolution for high frequency components.

II. DISCRETE WAVELET TRANSFORM

The discrete wavelet transform is computed at scales $s = a^j$. By choosing a = 1/2, we can define a discrete wavelet set $\{\varphi_{j,k}(x)\}$ where

$$\varphi_{j,k}(x) = 2^{j/2}\varphi(2^j x - k) \qquad (1)$$

For all $j,k \in$ Z and $\varphi(x) \in L^2R$. And we can also write the scaling function as

$$\emptyset_{j,k}(x) = 2^{j/2}\emptyset(2^j x - k) \qquad (2)$$

Here, k determines the position of $\varphi_{j,k}(x)$and $\emptyset_{j,k}(x)$ along the x-axis; and $2^{j/2}$ controls their height or amplitude. By choosing the scaling function $\emptyset(x)$ wisely, $\{\emptyset j,k(x)\}$ can be made to span L^2R. If the function being expanded is a sequence of numbers, like samples of a continuous function $f(x)$, the resulting coefficients are called the discrete wavelet transform (DWT) of $f(x)$. By applying the principle of series expansion, the DWT coefficients of $f(x)$, are defined as

$$W\phi(jo,k) = \frac{1}{\sqrt{M}}\sum_x f(x)\emptyset_{jo},k(x) \qquad (3)$$

$$W\varphi(j,k) = \frac{1}{\sqrt{M}}\sum_x f(x)\varphi j,k(x) \qquad (4)$$

For $j \geq jo$ and the parameter M is a power of 2 which range from 0 to *j-1*. The function $f(x)$ now can be expressed as

$$f(x) = \frac{1}{\sqrt{M}}\sum_k W\phi(jo,k)\phi jo,k(x) \qquad (5)$$
$$+\frac{1}{\sqrt{M}}\sum_{j=jo}^{\infty}\sum_k \mathrm{W\,\varphi(j,k)\varphi j,k(x)}$$

where $\frac{1}{\sqrt{M}}$ acts as a normalizing factor.

The DWT transform divides an image into low and high frequency bands and it represents first order DWT. The low-low (LL) band at the coarsest scale and low-high (LH), high-low

(HL), high-high (HH) spatial frequency bands at different scales.

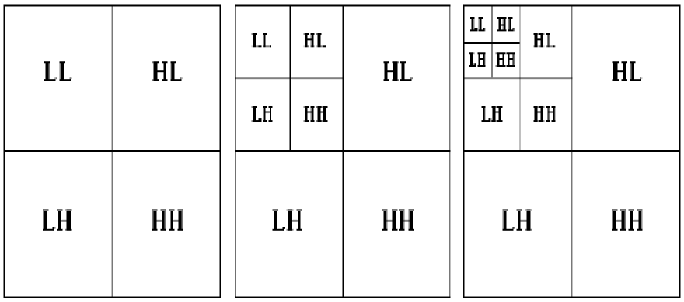


Fig.1 Wavelet Transform (a) single level decomposition, (b) two level decomposition, (c) three level decomposition

The LL band contains less spatial resolution and also contains the approximation detail of the original image. The other sub-part of image shows the detailed information. Decomposition can be done at four different stages. At first stage of decomposition there are four frequency bands. At next stages, decomposition is applied to low-low band of current decomposition that forms recursive decomposition. Finally, the Nth stage will have 3N+1 and one low-low frequency band.

III. WAVELET TRANSFORMS IN 2D

The two-dimensional wavelet transforms are slightly different to one-dimensional ones. One can easily extend it by simply multiply the one-dimensional scaling and wavelet functions. Wavelet transform in two dimensions is used in image processing. For two-dimensional wavelet transform, we need one two-dimensional scaling function, $\emptyset(x, y)$ and three two-dimensional wavelet functions, $\varphi^H(x, y)$, $\varphi^V(x, y)$ and $\varphi^D(x, y)$. Each is the product of a one-dimensional scaling function $\emptyset$ and corresponding wavelet φ. These are shown as follows:

$$\emptyset(x, y) = \emptyset(x)\emptyset(y) \qquad (6)$$

$$\varphi^H(x, y) = \varphi(x)\emptyset(y) \qquad (7)$$

$$\varphi^V(x, y) = \emptyset(x)\varphi(y) \qquad (8)$$

$$\varphi^D(x, y) = \varphi(x)\varphi(y) \qquad (9)$$

For image processing, these functions measure the variation of intensity for the image along different directions: φ^H measures variations along columns φ^V measures variations along rows, and φ^D measures variations along diagonals. The scaling function $\emptyset$ gives the approximation as same as the one-dimensional one. When the scaling function and wavelet functions are given, extension of the one-dimensional DWT to two-dimensions is straightforward. We first define the basis functions:

$$\emptyset_{j,m,n}(x, y) = 2^{j/2}\emptyset(2^j x - m, 2^j y - n) \qquad (10)$$

$$\varphi^i_{j,m,n}(x, y) = 2^{\frac{j}{2}}\varphi^i(2^j x - m, 2^j y - n) \qquad (11)$$

$$where\ i = \{H, V, D\}$$

where the index *i* defines the direction of the wavelet functions. The discrete wavelet transform function $f(x, y)$of size M X N is

$$W_\emptyset(j_o, m, n) = \frac{1}{\sqrt{M}}\sum_{x=0}^{M-1}\sum_{y=o}^{N-1} f(x, y)\emptyset_{j_0,m,n}(x, y) \quad (12)$$

$$W_\varphi(j, m, n) = \frac{1}{\sqrt{M}}\sum_{x=0}^{M-1}\sum_{y=o}^{N-1} f(x, y){\varphi^i}_{j,m,n}(x, y) \quad (13)$$

$$where\ i = \{H, V, D\}$$

Similarly, the variable j_0 is an arbitrary starting scale and $W_\emptyset(j_{0,}m, n)$define the approximation of $f(x, y)$. Moreover, $f(x, y)$ can be obtained by two-dimensional inverse discrete wavelet transform defined by

$$f(x, y) = \frac{1}{\sqrt{MN}}\sum_m\sum_n W_\emptyset(j_0, m, n)\emptyset j_0, m, n(x, y) + \frac{1}{\sqrt{MN}}\sum_{i=H,V,D}\sum_{j=j_0}^{\infty}\sum_m\sum_n W^i_\varphi(j, m, n)\varphi^i_{j,m,n}(x, y) \qquad (14)$$

IV. COMPRESSION STEPS

1. Digitize the source image into a signal s, which is a string of numbers.
2. Decompose the signal into a sequence of wavelet coefficients w.
3. Use threshold to modify the wavelet coefficients from w to w'.
4. Use quantization to convert w' to a sequence q.
5. Entropy encoding is applied to covert q into a sequence e.

V. QUALITY EVAULATION OF THE COMPRESSED IMAGE

A. *Mean Square Error (MSE)* :

Mean square error is one of the most commonly used error projection method where, the error value is the value difference between the actual data and the resultant data. The mean of the square of this error provides the error or the actual difference between the expected/ideal results to the obtained or calculated result.

$$MSE = {}^{1}/_{mn}\sum_{i=1}^{m}\sum_{=1}^{n}\left(A_{ij} - B_{ij}\right)^2 \qquad (15)$$

where m is the height of the Image implying the number or pixel rows, n is the width of the image, implying the number of pixel columns. Aij being the pixel density values of the perfect image. Bij being the pixel density values of the compressed image. Here, the calculation is performed at pixel level. A total of m*n pixels are to be considered. Aij will be the pixel density value of the perfect image and Bij being that of the compressed image. The difference between the pixel density of the perfect image and the compressed image is squared and the mean of the same is the considered error. MSE value will be 0 if both the images are identical.

B. Root Mean Square Error (RMSE)

RMSE events the quantity of change per pixel in an image due to the processing. It is given by:

$$RMSE = \sqrt{{}^{1}/_{mn} \sum_{i=1}^{m} \sum_{=1}^{n} \left(A_{ij} - B_{ij}\right)^2} \quad (16)$$

where

m - no. of rows
n - no. of columns
i - pixel row index
j - pixel column index
A - perfect image
B - compressed image.

C. Peak signal to noise ratio (PSNR)

PSNR is an engineering term for the ratio between the maximum possible power of a signal and the power of undignified noise that affect the fidelity of its image.

$$PSNR = 20log_{10}\left[\frac{L^2}{RMSE}\right] \quad (17)$$

where L - no. of grey levels in an image.

D. Compression Ratio (CR)

Compression ratio is defined as the ratio between the original image size and compressed image size

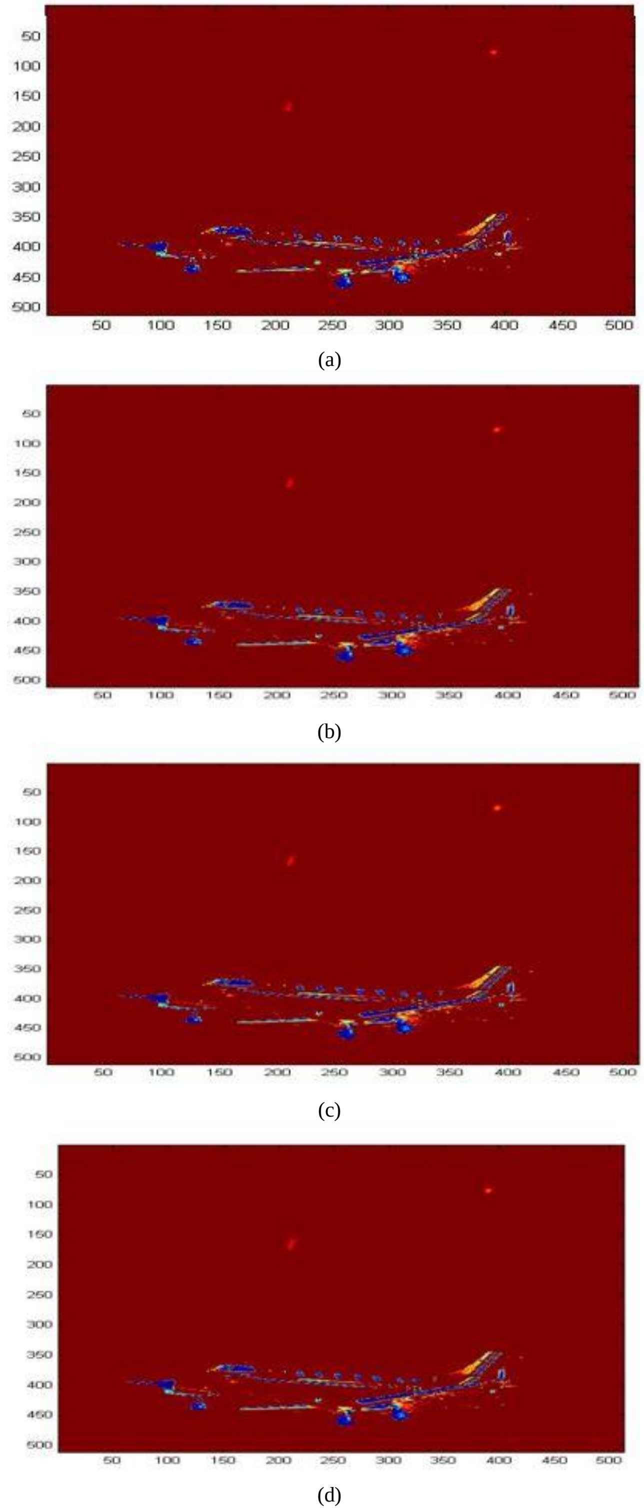
$$CR = \frac{Uncompressed\ image\ size}{Compressed\ image\ size} \quad (18)$$

E. Energy Retained (ER) :

$$ER = \frac{||x(n)||^2}{||r(n)||^2} * 100 \quad (19)$$

$||x(n)||$ is the norm of the original signal and $||r(n)||$ is the norm of the reconstructed signal.

VI. IMPLEMENTATION



(a) (b) (c) (d)

Fig2 (a) original image, (b) compressed image using haar wavelet, (c)compressed image using db wavelet, (d) compressed image using bior wavelet.

A. Analysis table:

TABLE 1
IMAGE QUALITY EVALUATION FOR COMPRESSED IMAGE USING DIFFERENT WAVELETS AT DIFFERENT LEVELS OF DECOMPOSITION

DWT	Transform		MSE	PSNR	ER	CR
	wavelet	level				
haar	haar	2	10.329	37.990	90.052	99.995
	haar	4	1.778	45.629	94.756	99.995
db	db2	2	7.652	39.292	90.997	99.996
	db4	4	2.236	44.634	95.941	99.997
bior	bior2.2	2	5.000	41.140	91.386	99.997
	bior4.4	4	2.233	44.138	96.360	99.997

VII. CONCLUSION

The Discrete Wavelet Transform provides a multi-resolution representation of images. In traditional algorithms, spatial domain has limitations such as transmission of codebook along with the image which is higly complex and computationally intensive, hence hinders real time implementation. Image data compression in previous two decade achieves substantial progress. In overview study, every new approach gives better performance compare to previous methods. There are two standards introduced in image data compression: JPEG and JPEG2000, where both perform as lossless or lossy image compression respect to different algorithms. Image compression used at different images like medical images, natural image, artificial images and satellite image etc. basically data compression most applicable when we need to transmit or store a huge amount of data. In this report haar, daubechies and biorthogonal wavelet transforms are used to compress an image. Results on the bases of Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR), Energy Retained (ER) and Compression Ratio (CR) have been calculated and a comparison analysis is also displayed for the same. Experimental results demonstrate that BIOR wavelet shows best result among the used wavelets.

VIII. FUTURE SCOPE

At present, image compression is more concentrated on two-dimensional images but in future, more emphasis will be on three-dimensional images. New approaches are being proposed for progressive work in term of feature preserve with compression rate for image data compression.

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